

Mathematics 601, MWF 1:00, E 206

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Topics

The Classical Groups (a class I will consider rather generally, and in which I will include related objects) are a family that arises – from different intentions – in many areas of mathematics. This course aims to give an introduction to these groups and the manifold ways how they can be constructed and studied.

If you are interested to work in algebra or combinatorics, or need to work with groups, these are topics you likely will find interesting and useful.

Caveat: I am an algebraist/combinatorialist, not a geometer and my viewpoint therefore will be less geometric than that of others.

Grades

Will be based on homework (there will be four homework assignments) and course participation;

Prerequisites: In general, you should have taken the M566/567 sequence or an equivalent course. (There are just a handful of facts from 567 we shall need and I will repeat these.) In fact we will formally require little material beyond 369 and 466, but proceed at the pace of a 600-level mathematics course.

Textbooks and Literature

- [Art57] E. Artin. *Geometric algebra*. Interscience Publishers, Inc., New York-London, 1957.
- [BB10] Alexandre V. Borovik and Anna Borovik. *Mirrors and reflections*. Universitext. Springer, New York, 2010. The geometry of finite reflection groups.
- [Cam99] Peter J. Cameron. *Permutation Groups*, volume 45 of *London Mathematical Society Student Texts*. Cambridge University Press, 1999.
- [Cam00] Peter J. Cameron. Notes on classical groups. http://www.maths.qmul.ac.uk/~pjc/class_gps/, 2000.
- [Car65] R. W. Carter. Simple groups and simple Lie algebras. *J. London Math. Soc.*, 40:193–240, 1965.
- [Car72] Roger W. Carter. *Simple groups of Lie type*. John Wiley & Sons, London-New York-Sydney, 1972. Pure and Applied Mathematics, Vol. 28.
- [CCN+85] J[ohn] H. Conway, R[obert] T. Curtis, S[imon] P. Norton, R[ichard] A. Parker, and R[obert] A. Wilson. *ATLAS of finite groups*. Oxford University Press, 1985.

- [Dic58] Leonard Eugene Dickson. *Linear groups: With an exposition of the Galois field theory.* with an introduction by W. Magnus. Dover Publications Inc., New York, 1958.
- [DM96] John D. Dixon and Brian Mortimer. *Permutation Groups*, volume 163 of *Graduate Texts in Mathematics*. Springer, 1996.
- [GB85] L. C. Grove and C. T. Benson. *Finite reflection groups*, volume 99 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1985.
- [Gro02] Larry C. Grove. *Classical groups and geometric algebra*, volume 39 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2002.
- [Hum90] James E. Humphreys. *Reflection groups and Coxeter groups*, volume 29 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1990.
- [MT11] Gunter Malle and Donna Testerman. *Linear algebraic groups and finite groups of Lie type*, volume 133 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2011.
- [Rom05] Steven Roman. *Advanced linear algebra*, volume 135 of *Graduate Texts in Mathematics*. Springer, New York, second edition, 2005.
- [Spr98] T. A. Springer. *Linear algebraic groups*, volume 9 of *Progress in Mathematics*. Birkhäuser Boston Inc., Boston, MA, second edition, 1998.
- [Tay92] Donald E. Taylor. *The geometry of the classical groups*, volume 9 of *Sigma Series in Pure Mathematics*. Heldermann Verlag, Berlin, 1992.
- [Wey39] Hermann Weyl. *The Classical Groups. Their Invariants and Representations*. Princeton University Press, Princeton, N.J., 1939.
- [Wil09] Robert A. Wilson. *The finite simple groups*, volume 251 of *Graduate Texts in Mathematics*. Springer-Verlag London Ltd., London, 2009.

While there are many books that cover some aspects of the material, I have not found a single source. The following is a list of books and articles I have been using in preparation for the course. Some are available freely on the Web. (Note that the CSU library allows (via www.springerlink.com) the download of many of the Springer/Birkhäuser textbooks.) Some (in particular [Car72]) are alas priced beyond obscene, or hard [Tay92] to obtain.

Some of these books border on research monographs and do not make for easy reading. The aim of the course is not to have you understand all the material in these tomes, but to enable you to use the different view points.

Amongst these books, [Art57, Dic58, Wey39] are classical texts that started to define the subject. [DM96, Cam99] describe the theory of permutation groups and group actions. [BB10, GB85, Hum90] are about the related class of Coxeter Groups and Weyl Groups. [Wil09] is the purest group theoretic book and is focussed on the groups as examples of simple groups. [Car65, Car72] take the point of view of Groups of Lie Type. [MT11, Spr98] start with algebraic groups, i.e. a consideration of the groups as varieties. A modern geometric aspect is taken by [Cam00, Gro02, Tay92]. A terse summary of facts can be found in the introduction of [CCN+85]. The required linear algebra can be found in [Rom05].

I expect that [Wil09, DM96, GB85, Car72, MT11, Tay92] will be my main sources for the different aspects.

I wish you success with this course and all the best for the coming semester.