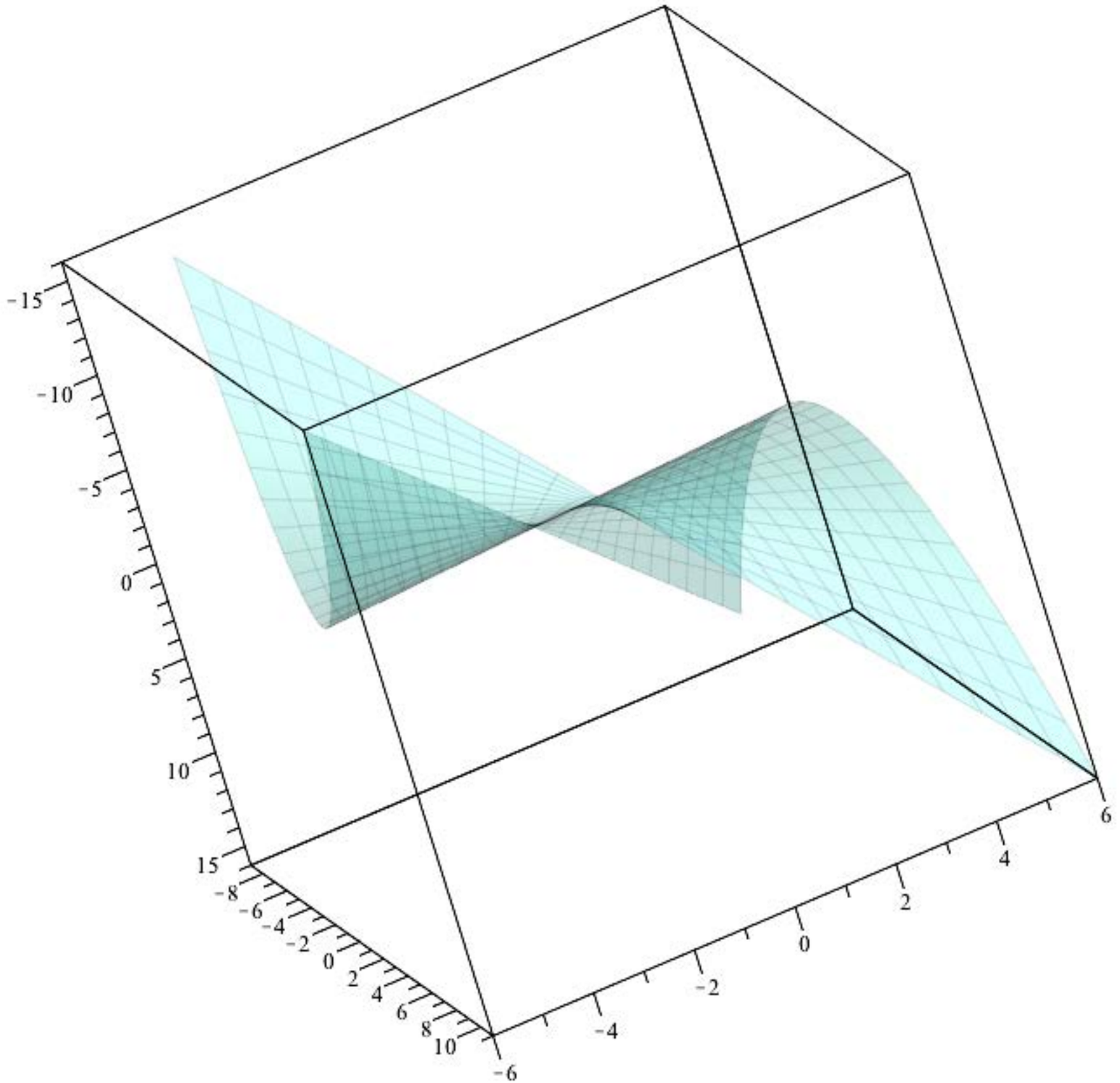
-15
-10
-5
0
5
10
15
-8
-6
-4
-2
0
2
4
6
8
10
-6
-4
-2
0
2
4
6

```
gap> R:=PolynomialRing(Rationals,
["t","u","x","y","z"]);
Rationals[t,u,x,y,z]
gap> AssignGeneratorVariables(R);
#I  Assigned the global variables [ t, u, x, y,
z ]

gap> I:=[x-(t+u),y-(t^2+2*t*u),z-(t^3-3*t^2*u)];
[ -t-u+x, -t^2-2*t*u+y, -t^3+3*t^2*u+z ]

gap> SetInfoLevel(InfoGroebner,3);
gap>
ReducedGroebnerBasis(I,MonomialLexOrdering());
#I  Spol(2,1)=t*u+t*x-y
#I  reduces to -u^2+x^2-y
#I  |bas|=4, 7 pairs left
#I  Spol(3,1)=-4*t^2*u+t^2*x-z
#I  reduces to -10*u*x^2+10*x^3+4*u*y-9*x*y-z
#I  |bas|=5, 11 pairs left
#I  Pair (3,2) avoided
#I  Pair (4,1) avoided
#I  Pair (4,2) avoided
#I  Pair (4,3) avoided
#I  Spol(4,4)=0
#I  Pair (5,1) avoided
#I  Pair (5,2) avoided
#I  Pair (5,3) avoided
#I  Spol(5,4)=-10*u*x^3+10*x^4-4*u^2*y
+9*u*x*y-10*x^2*y+u*z
#I  reduces to 5*u*x*y-5*x^2*y+u*z+x*z+4*y^2
#I  |bas|=6, 8 pairs left
#I  Spol(5,5)=0
#I  Pair (6,1) avoided
#I  Pair (6,2) avoided
```

```
#I  Pair (6,3) avoided
#I  Spol(6,4)=5*u*x^2*y-5*x^3*y-u^2*z-
u*x*z-4*u*y^2+5*x*y^2
#I  reduces to -u*x*z-2*u*y^2-x^2*z
+1/2*x*y^2+1/2*y*z
#I  |bas|=7, 10 pairs left
#I  Spol(6,5)=-10*u*x*z-20*u*y^2-10*x^2*z
+5*x*y^2+5*y*z
#I  Spol(6,6)=0
#I  Pair (7,1) avoided
#I  Pair (7,2) avoided
#I  Pair (7,3) avoided
#I
Spol(7,4)=2*u^2*y^2+u*x^2*z-1/2*u*x*y^2+x^3*z-1/
2*u*y*z-x*y*z
#I  reduces to 2*x^3*z
+3/2*x^2*y^2-9/5*x*y*z-8/5*y^3-1/10*z^2
#I  |bas|=8, 12 pairs left
#I
Spol(7,5)=20*u*x*y^2+20*x^3*z-5*x^2*y^2+4*u*y*z-
14*x*y*z-z^2
#I  Spol(7,6)=-10*u*y^3-10*x^2*y*z
+5/2*x*y^3+u*z^2+x*z^2+13/2*y^2*z
#I  reduces to -10*u*y^3-10*x^2*y*z
+5/2*x*y^3+u*z^2+x*z^2+13/2*y^2*z
#I  |bas|=9, 19 pairs left
#I  Spol(7,7)=0
#I  Pair (8,1) avoided
#I  Pair (8,2) avoided
#I  Pair (8,3) avoided
#I  Pair (8,4) avoided
#I  Spol(8,5)=-15*u*x^2*y^2-20*x^4*z+10*u*x*y*z
+16*u*y^3+18*x^2*y*z+u*z^2+2*x*z^2
#I  Pair (8,6) avoided
```

```
#I  Pair (8,7) avoided
#I  Spol(8,8)=0
#I  Pair (9,1) avoided
#I  Pair (9,2) avoided
#I  Pair (9,3) avoided
#I
Spol(9,4)=10*u*x^2*y*z-5/2*u*x*y^3+10*x^2*y^3-
u^2*z^2-u*x*z^2-13/2*u*y^2*z-10*y^4
#I  Spol(9,5)=100*x^4*y*z
+75*x^3*y^3-10*u*x^2*z^2+40*u*y^4-10*x^3*z^2-65*
x^2*y^2*z-90*x*y^4-\
10*y^3*z
#I
Spol(9,6)=-50*x^3*y*z-75/2*x^2*y^3+5*u*x*z^2+10*
u*y^2*z+5*x^2*z^2+85/2*x*y^2*z+40*y^4
#I  Pair (9,7) avoided
#I  Pair (9,8) avoided
#I  Spol(9,9)=0
[ x^3*z+3/4*x^2*y^2-9/10*x*y*z-4/5*y^3-1/20*z^2,

u*y^3+x^2*y*z-1/4*x*y^3-1/10*u*z^2-1/10*x*z^2-13
/20*y^2*z,
  u*x*z+2*u*y^2+x^2*z-1/2*x*y^2-1/2*y*z, u*x*y-
x^2*y+1/5*u*z+1/5*x*z+4/5*y^2,
  u*x^2-x^3-2/5*u*y+9/10*x*y+1/10*z, u^2-x^2+y,
t+u-x ]
gap>
```