

D&F;1.7: 6

D&F;2.1: 2ad,3,5,8,16

D&F;2.2: 2,6,11

1) Let G be a group. G acts on itself by *conjugation*, that is by

$$(h, g) \mapsto h^g := g^{-1}hg$$

Show that this satisfies the axioms of a group action.

In GAP, you can construct the homomorphism induced by a group action in the following way:

```
ActionHomomorphism(G, A, actionfun);
```

where G is a group, A is the domain and `actionfun` indicates the action. The Image of such a homomorphism is the resulting permutation group, `Kernel` is the kernel of the action.

Predefined actions (possible values for `actionfun`) are:

`OnPoints` Permutation group on points, `Matrix` group on vectors, `Group` on its elements or subgroups by conjugation.

`OnRight` Right multiplication (group on its elements)

`OnTuples` A permutation group on tuples of points

`OnSets` A permutation group on sets of points (the sets must be sorted)

For example

```
g:=SymmetricGroup(3);
ActionHomomorphism(g, AsList(g), OnRight);
ActionHomomorphism(g, [[1,2],[1,3],[2,3]], OnSets);
```

gives the action of S_3 on itself by right multiplication, respectively on sets of 2 points.

In the same syntax, you can specify:

```
Stabilizer(g, point, actionfun);
```

There also are `Centralizer(G, elm)`, `Centre(G)` and `Normalizer(G, H)`.

2) (GAP) Let $G = S_5$ (`SymmetricGroup(5)`). Compute the centralizers for one element of each possible cycle type in G .

3) (GAP) Take the elements of S_5 that are 5-cycles

```
Filtered(AsList(g), i->CycleStructurePerm(i)=[, , , 1]);
```

and form the different subgroups generated by them.

Let S_5 act on the set A of these subgroups by conjugation (OnPoints). What is the image of this action? Is the action faithful?

Deduce that S_5 is isomorphic to two different subgroups of S_6 .

4) (GAP) Take the group of rotations and reflections of a cube (Problem 3 on the last homework) and let it act on the *diagonals* of the cube (represent them as sets of points). What Image group do you get?

Describe the elements of the kernel of this action.