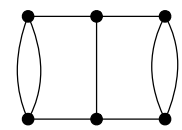
Mathematics 502 **Homework (due Feb. 22)** A. Hulpke

20) Determine the number of different (up to rotation/reflection) necklaces with 6 pearls, if every pearl could be colored in one of three colors.

21) A hypothetical chemical compound, bambuzole, consists of 6 carbon atoms connected as depicted, with each carbon atom connected to either a single hydogen atom (H) or an hydroxide (OH) group. Determine the number of different bambuzoles that have four H groups and two OH groups.



22) This problem aims to describe the cycle structure of a permutation $g \in S_n$ when acting on the set Ω consisting of the $\binom{n}{2}$ pairs of points. We need to distinguish the cases of the the two points in the pair lying in the same cycle of g or not.
a) Show that g has $\gcd(a,b)$ cycles on Ω, each of length $\operatorname{lcm}(a,b) = \frac{ab}{\gcd(a,b)}$, of pairs with one point coming from a cycle of length a and the second point coming from a disjoint cycle of length b.
b) Show that if g has a cycle C of length a, a odd, there are $(a-1)/2$ cycles of length a on pairs of points both from C.
c) Show that if g has a cycle C of length a, a even, then g has on pairs of points from C one cycle of length $m/2$ (pairs at distance exactly $a/2$), as well as $(a-2)/2$ cycles of length a.

23) (Continuation of 22)
a) Determine the cycle structure of the elements of S_6 (there are 11 partitions) on the $\binom{6}{2} = 15$ sets of pairs of points.
b) Determine the total number of (unlabelled) graphs on 6 vertices.

24) We denote by $p_k(n)$ the number of partitions of n into exactly k cells. Show that for an arbitrary integer x (and thus also for x as a real variable) we have that

$$\sum_{k=0}^{n} p_k(n) x^k = x(x+1)\cdots(x+n-1)$$

Hint: Consider a set X with x elements and consider the functions from $\{1,\ldots,n\}$ to X. Then consider the number of orbits of S_n on these functions, once by the counting lemma, and once by a counting interpretation from chapter I.