Mathematics 502 **Homework (due Feb. 22)** A. Hulpke

16) Let $n = a + b$ with $a < b$. Show that $S_a \times S_b$ is a maximal subgroup of S_n.
(**Hint:** Consider the action of S_n on sets of a points and show that this action is primitive.)

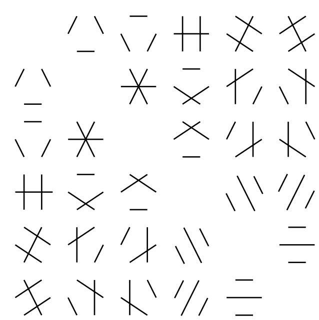
17) Let G be a group acting transitively on the set Ω. We define an action of G on $\Omega \times \Omega$ by setting $(\delta, \omega)^g := (\delta^g, \omega^g)$. Find a bijection between the orbits (not the points in the orbits!) of G on $\Omega \times \Omega$ and the orbits of $\mathrm{Stab}_G(\omega)$ on Ω.

18) For subgroups $H, K \leq G$ and $g \in G$ we denote the *double coset of H and K with representative g* by

$$HgK = \{hgk \mid h \in H, k \in K\}$$

a) Show that the double cosets for H and K give a partition of G.
b) Show that HgK is a union of right cosets of H.
c) Suppose that G acts on Ω and that $H = \mathrm{Stab}_G(\omega)$ for some $\omega \in \Omega$. Show that the double cosets HgK correspond to the orbits of K on ω^G.

19) Let $n = 6$. There are $\binom{6}{2} = 15$ pairs of points which we can consider as edges in the complete graph. A *factor* is a set of 3 disjoint edges. A *factorization* is a partition of the 15 edges into five factors.
a) Show that two disjoint factors are contained in a unique factorization.
b) Show that there are exactly six factorizations. (Compare the logo for the Combinatorics Seminar shown.)



c) Define an action of S_6 on factorizations. This gives a homomorphism $S_6 \to S_6$. Show that it is bijective.
d) Show that the automorphism constructed in c) is an outer automorphism. (Consider for example the image of $(1, 2)$ under the map).
(We thus conclude that S_6 has two nonequivalent transitive permutation actions on 6 points. One can show that this phenomenon is unique to the number 6.)