11) Let $\mathcal{B}$ be a block system for the action of G on Ω. Show that the block $B \in \mathcal{B}$ containing $\omega \in \Omega$ must be the union of orbits of $\mathrm{Stab}_G(\omega)$.

12) Let G be a finite simple group. Show that there exists a degree n and a primitive permutation group $P \le S_n$ such that $P \cong G$.

13) Show that the symmetry group of the regular octahedron is the wreath product $S_2 \wr S_3$, with the imprimitive action on the six vertices and a product action on the 8 faces. Is this group isomorphic to S_4?

14) Let $G = \mathtt{TransitiveGroup}(12, 50)$

$$= \langle (1,8)(2,7)(3,12)(4,5)(6,9)(10,11), (1,11,9,7,5,3)(2,6,10)(4,8,12) \rangle .$$

This group has order 96 and

$$\mathrm{Stab}_G(1) = \langle (3,9)(6,12), (2,8)(5,11), (2,8)(3,9)(4,10) \rangle$$

a) Find all block systems (i.e. all G-invariant congruences on $\{1, \ldots, 12\}$.)
Hint: In a nontrivial block system there must be a point that shares a block with 1. Try out possibilities for adding points (using problem 11 to reduce the number of choices) and use the fact that the images form a partition to grow blocks and their images. Iterate to get nonminimal blocks.
b) Find all subgroups $\mathrm{Stab}_G(1) \le S \le G$.

15) The group S_n acts on its elements by conjugation $g^h := h^{-1}gh$, the stabilizer of g under this action is called the *Centralizer*

$$C_{S_n}(g) = \{h \in S_n \mid h^{-1}gh = g\} = \{h \in S_n \mid gh = hg\}.$$

Suppose that $g \in S_n$ is a permutation with cycle structure $1^{a_1}2^{a_2}\cdots n^{a_n}$. Show that $C_{S_n}(g)$ has the structure

$$(Z_1 \wr S_{a_1}) \times (Z_2 \wr S_{a_2}) \times \cdots \times (Z_n \wr S_{a_n})$$

where Z_i denotes a cyclic group of order i (acting as an i-cycle).