60) Let $\mathcal{C}$ be a perfect linear e-error correcting code over $\mathbb{F}_q$. Show that the weight enumerator of $\mathcal{C}$ is uniquely determined by the code parameters.

61) Let $g(x) = 1 + x + x^3$ be a polynomial with coefficients in $\mathbb{F}_2$.
a) Show that $g(x)$ is a factor of $x^7 - 1$ in $\mathbb{F}_2[x]$.
b) The polynomial $g(x)$ is the generating polynomial for a cyclic code $\mathcal{C}$. Find a generating matrix for $\mathcal{C}$ and a parity check matrix H for $\mathcal{C}$ and show that $\mathcal{C}$ is equivalent to a Hamming code.

62) Let $g(x)$ be the generating polynomial for a cyclic code $\mathcal{C}$ of length n and $h(x) = x^n - 1/g(x) = b_0 + b_1 x + \cdots + b_{l-1}x^{l-1} + x^l$. Show that the dual code $\mathcal{C}^\perp$ is cyclic with generating polynomial $1/b_0(1 + b_{l-1}x + \cdots + b_1 x^{l-1} + b_0 x^l)$. (The factor $1/b_0$ is included to make the highest nonzero coefficient be 1.)