45) Let n be an integer and $m = n^2$. Show that an $m \times m$ Sudoku problem can only have a unique solution if all of the following conditions are fulfilled:

- It contains at least $m - 1$ numbers
- It contains numbers in $n(n-1)$ different rows
- It contains numbers in $n(n-1)$ different columns

(It has been proven in 2012 that there is no 9×9 Sudoku with less than 17 given numbers that has a unique solution.)

46) Find, using an implementation of the LLL algorithm, a combination of the numbers

$$276, 1768, 1993, 2536, 4251, 4884, 5020, 5347, 7401, 9072$$

that sums up to 33164.

47) Let $J = 1^{n \times n}$ and I the identity matrix. Show that

$$\det(xI + yJ) = (x + yn)x^{n-1}.$$

Hint: Show that the eigenvalues of the matrix are x (with multiplicity $n-1$) and $x + ny$.

48) For $n > 2$ let X be the points of $PG_n(q)$ and $\mathcal{B}$ the i-flats (i.e. the $i+1$ dimensional subspaces of $\mathbb{F}_q^{n+1}$). What are the parameters of such a design?

49) a) Why can't there be a $4-(8,6,1)$ design?
b) Suppose that $k-1$ is a prime. Show that there are $2-(v,k,\lambda)$ designs only if $k-1 \mid v-1$ or $k-1 \mid \lambda$.