Mathematics 502 **Final (100 points)** 5/12/21, 4.00pm

Points (leave blank)				
1	2	3	4	$\sum$

Name:

(clearly, please)

This exam is my own work. Sources (apart from class notes) are indicated. I have not given, received, or used any unauthorized assistance.

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Signature

Notes

- Put your name on this cover sheet and sign it.
- Work the exam in a single block of 2 hours.
- All problems carry equal weight.
- You are not permitted to discuss the question with others.
- Nor may you post exam questions in public fora.
- A description of how you solved the problem, respectively justification of the steps taken, is a crucial part of every solution.
- You are permitted to use class notes and any publication (book, journal, web page). Results which are quoted from a publication (apart from the course notes and your lecture notes) must be indicated.
- You may use a computer unless this renders the problem trivial.

Mathematics 502 **Exam Questions** A. Hulpke

1) Determine the cycle index for the action of A_5 on the 10 two-element subsets of $\{1, \ldots, 5\}$.

2) Construct a [15,11,3] BCH code. E.g. Describe how to find the generator matrix, or the check matrix, but you do not need to write out the full matrix. Explain why it has the specified parameters. Note: Over $\mathbb{F}_2$ we have

$$x^{15} - 1 = (x+1) \cdot (x^2 + x + 1) \cdot (x^4 + x + 1) \cdot (x^4 + x^3 + 1) \cdot (x^4 + x^3 + x^2 + x + 1)$$

3) For a projective plane of order n we form an incidence matrix $A \in \{0,1\}^{m \times m}$ (with $m = n^2 + n + 1$) whose rows correspond to lines and whose columns correspond to points. Let $\mathcal{C}$ be the $\mathbb{F}_2$-code generated by the rows of this matrix.
a) Show that for odd n this code $\mathcal{C}$ consists of all the words of even weight.
b) Now consider $n \equiv 2 \pmod 4$. We form an extended code $\mathcal{C}'$ by adding to every code word one parity bit. Show that $\mathcal{C}' \subset \mathcal{C}'^{\perp}$.

4) Let $\{A_1, \ldots, A_k\}$ a set of MOLS of order n for $k \leq n - 2$. We define a graph $\Gamma = (X, E)$ with $X = \{(i,j) \mid 1 \leq i, j \leq n\}$ and an edge given between (i,j) and (a,b) if one of the following conditions holds:

- $i = a$
- $j = b$
- For one of the latin squares A_x, its (i,j) entry and its (a,b) entry are equal.

Show that the graph defined by k MOLS of order n is strongly regular with parameters

$$(n^2, (n-1)(k+2), n - 2 + k(k+1), (k+1)(k+2)).$$