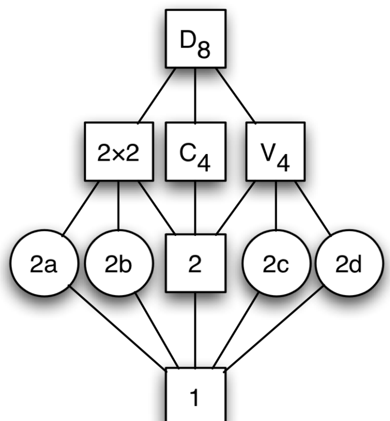
38) Show that for a poset P the set $I(P)$ is an algebra (a ring and a vector space with compatible operations) with one, that is, show (the rest is trivial) that the multipliction is associative and that the distributive laws hold.

39) Let P be a finite poset, $|P| = n$, and assume we arrange (embed into total ordering) the elements of P so that $x_i \le x_j$ implies that $i \le j$. Let K be a field and

$$A = \left\{ M = (m_{i,j}) \in K^{n\times n} \mid m_{i,j} = 0 \text{ whenever } x_i \nleq x_j \right\}.$$

a) Show that A is closed under addition and matrix multiplication.
b) Show that $I(P)$ (over the field K) is isomorphic to A.

40) Determine $\mu(0,1) = \mu(\langle 1\rangle, D_8)$ for the lattice of subgroups of D_8, the dihedral group of order 8, given on the side.



41) a) Let a,b be elements of a poset P. Prove that

$$\mu(a,b) = \sum_{i\ge 0} (-1)^i c_i,$$

where c_i is the number of chains $a = x_0 < x_1 < \cdots < x_i = b$. (Hint: Show that the right hand side fulfills the defining property of μ.)
b) For a poset P, let P^* be the poset obtained by reversing the order relation (that is $x \le_{P^*} y$ iff $y \le_P x$). Conclude that $\mu_{P^*}(a,b) = \mu_P(b,a)$.

41$\frac{1}{2}$) (For those who have seen algebraic topology). For a poset P with a global minimal element 0 and a global maximal element 1, we define a simplicial complex $\Delta(P)$, called the *order complex*, as follows: The elements of P are the vertices of $\Delta(P)$, the chains of P are the faces. Show that $\mu_P(0,1) = \chi(\Delta(P)) - 1$, where χ is the ordinary Euler characteristic.