Mathematics 501 **Homework (due Sep. 2)** A. Hulpke

1) According to J.L. Borges, *The Library of Babel* (in *Ficciones*, 1944, here quoted from `http://www.analitica.com/bitblioteca/jjborges/library_babel.asp`):

> is composed of an indefinite and perhaps infinite number of **hexagonal galleries**, with vast air shafts between, surrounded by very low railings. From any of the hexagons one can see, interminably, the upper and lower floors. The distribution of the galleries is invariable. **Twenty shelves, five long shelves per side, cover all the sides except two**; their height, which is the distance from floor to ceiling, scarcely exceeds that of a normal bookcase. One of the free sides leads to a narrow hallway which opens onto another gallery, identical to the first and to all the rest. [...]
>
> There are five shelves for each of the hexagon's walls; **each shelf contains thirty-five books of uniform format; each book is of four hundred and ten pages; each page, of forty lines, each line, of some eighty letters** which are black in color. [...] **The orthographical symbols are twenty-five in number**. [...] These examples made it possible for a librarian of genius to discover the fundamental law of the Library. This thinker observed that all the books, no matter how diverse they might be, are made up of the same [25] elements: the space, the period, the comma, the twenty-two letters of the alphabet. He also alleged a fact which travelers have confirmed: In the vast Library **there are no two identical books**.

a) Following the description as given by the bold sections; How many books does the library contain?
b) How many rooms does the library have?
c) The Earth has a land area of about 60,000,000 square miles. Assuming that each room is 100 square foot, how many stories high would one have to build to fit the library? (You may use 1mi ≈ 5000ft for convenience).

2) A web site uses passwords consisting of 4 symbols, chosen from (upper case) letters and digits.
a) Determine the number of passwords of length 4 that contain at least one digit. What percentage of all passwords is this? Does a policy of requiring a digit in each password improve security?
b) Determine the number of passwords of length 4 that contain exactly one digit.
c) Determine the number of passwords of length 4 that contain at most two (i.e. 0, 1, or 2) digits.

3) Prove that $\binom{n}{2} + \binom{n+1}{2} = n^2$ in two ways – once algebraically, and once using a combinatorial interpretation of counting objects.

4) A well dressed giraffe is supposed to wear four scarves. There are 7 possible scarf colors and it is a serious social mistake to have three or more scarves in the same color. How many different, socially acceptable, outfits exist if the arrangement of the scarves is relevant, how many if the arrangement does not matter?

5) A graduate student creates slides for her masters' presentation and wants to use different colors on subsequent pages. There are 15 pages and 5 available colors.
a) How many different ways does she have to select colors?
b) The advisor recommends to use two colors on each slide, not repeating either. How many different ways of coloring exist?

6) Assuming you want to calculate bionomial coefficients on a computer that cannot handle numbers bigger that some absolute value (say 2^{16}), which of the following four methods is preferrable, and why?

a) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

b) $\binom{n}{k} = n(n-1)\cdots(n-k+1)/k!$

c) $\binom{n}{0} = 1, \binom{n}{k} = \binom{n}{k-1} \cdot \frac{n-k+1}{k}.$

d) $\binom{n}{0} = \binom{n}{n} = 1, \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ (Pascal's Triangle)