

**36)** Let  $G$  be a group. We have seen that  $G$  acts on  $G$  via the action  $\mu(x, g) := g^{-1}xg$ . The orbits under this action are called the *conjugacy classes* of  $G$ .

a) Show that every normal subgroup of  $G$  is a union of conjugacy classes.

b) Determine the conjugacy classes of  $S_5$  and their cardinalities, by counting the number of elements of each cycle structure. (You can use the GAP command

```
cl:=ConjugacyClasses(SymmetricGroup(5));  
List(cl,Size);
```

to verify the combinatorial counts of the cardinalities. However your answer should not be based on a computer calculation.)

c) Show that  $S_5$  can have normal subgroups only of orders 1, 60, 120.

**37)** Let  $G$  be a group and  $S \leq G$  with  $[G : S] = 3$ . Show that there must be  $N \triangleleft G$  with  $[G : N] \in \{2, 3\}$ .

**Hint:** Consider the action of  $G$  on the cosets of  $S$ . Show that its image must be equal to  $A_3$  or to  $S_3$ . In the second case consider the permutations that map to  $A_3 \leq S_3$ .

**38)** Determine a 2-Sylow subgroup and a 3-Sylow subgroup of  $S_4$ . How many Sylow subgroups (or which orders) are there in total in  $S_4$ ?

**39)** Let  $G$  be a group,  $S \leq G$  and  $p$  be prime with  $p \mid |S|$ . Let  $U$  be a  $p$ -Sylow subgroup of  $G$ . Show that if  $p$  does not divide  $[G : S]$ , then  $U$  is a  $p$ -Sylow subgroup of  $G$ .

**40)** Let  $G$  be a group such that  $|G| < 30$  and  $|G|$  is not prime. Show that there must be  $N \triangleleft G$  such that  $1 \neq |N| \neq |G|$ .

## Poster Session

The departmental poster session will be on Thursday, December 12, 9:30-11am in the student center. I would like to encourage everyone to participate. Posters grades can cover up to 15% of the total grade (beyond the homework component, this is relaxing the grading rules I initially announced). I will allow this to cover parts from both homework and midterm score. Producing a poster can only raise your grade. There also will be poster prizes ranging from \$50-\$150.

I would like to know whether you plan to produce a poster by November 1 (form on the bottom), and the title, I will need to see a PDF draft for the poster by December 2.

Posters need to cite their sources, but you are free to use whatever sources seem plausible. If you choose a topic I suggested on a previous homework I will be able to provide you with copies from relevant textbook sections, should you be interested in doing so.

**Poster format:** We will have 4' x 8' poster boards onto which students can tack on a poster; push pins will be provided. So, posters should fit within a 4' x 4' space. Note that the printers at the CNS poster lab have 36" wide rolls of paper. I don't know of free printing on campus for larger sizes.

**Poster template:** There will be no required style (layout, software, etc.) for the poster, but posters should have on it the authors name and the course number (MATH466).

A list of poster templates (in LaTeX format) are at

<https://www.sharelatex.com/templates/presentations>,

Example posters include:

<https://www.sharelatex.com/templates/presentations/beamerposter---flow-multicolumn>

<https://www.sharelatex.com/templates/presentations/wireless-foundations-conference-poster>

[https://www.sharelatex.com/templates/presentations/hmc-poster-\(clinic\)](https://www.sharelatex.com/templates/presentations/hmc-poster-(clinic))

<https://www.sharelatex.com/templates/presentations/cambridge-university-poster>

**Poster printing:** Students in the College of Natural Sciences can print posters without charge (up to "800 credits" are free) in the CNS computer lab. See <http://wp.natsci.colostate.edu/cnsit/az-e100-computer-lab/> and scroll down to the section on "Poster Printing". There is a sign up sheet in the CNS computer lab, and it is strongly recommended to sign up to reserve a time to print posters!

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### Poster Form

Name:

Poster Subject:

(This does not commit you to producing a poster.)

I would like material (photocopied book pages) about the chosen topic.