18) Describe (e.g. find a set of representatives of) the cosets of S_{n-1} in S_n.

19) a) Show that every group of prime order p is isomorphic to the cyclic group C_p.
b) Show that every group of order 4 is isomorphic to C_4 or to C_2. (Hint: What are the possible element orders? What follows for the multiplication table?)

20) Let $G = S_4 = \langle (1,2,3,4),(1,2)\rangle$. Show that $V_4 = \langle (1,2)(3,4),(1,3)(2,4)\rangle \lhd G$. Show that S_4/V_4 is not abelian.

21) a) Let G, H be groups. Show that the cartesian product (the set of pairs of elements) $G \times H$ becomes a group under component-wise operation, i.e. $(g_1,h_1)\cdot(g_2,h_2) := (g_1g_2,h_1h_2)$. (We call this the *direct product* of G and H.)
b) Show that the subsets $N_1 = \{(g,1_H) \mid g \in G\}$ and $N_2 = \{(1_G,h) \mid h \in H\}$ are both normal subgroups of $G \times H$ and that $N_1 \cap N_2 = \{1_{G\times H}\}$.
c) Let A be a group with $N, M \lhd A$ and $N \cap M = \{1\}$. Let

$$\varphi\colon A \to A/N \times A/M,\ a \mapsto (Na, Ma)$$

is an isomorphism.

22) Let $Q_8 = \langle (1,2,3,8)(4,5,6,7),(1,7,3,5)(2,6,8,4)\rangle$ be the group from homework problem 10.
a) Show that every subgroup in Q_8 is normal. (In fact it can be shown that any nonabelian group, in which every subgroup is normal, is a direct product of Q_8 with an abelian group.)
b) Show that Q_8 has a normal subgroup $N \lhd Q_8$ of order 2 such that Q_8/N is not cyclic, but that all subgroups of Q_8 of order 4 are cyclic.

23) Let G be a group and $H, K \le G$ with $H \le K \le G$ such that $[G{:}H]$ is finite.
a) Let $\{r_i\}$ a set of representatives of the cosets of K in G and $\{s_j\}$ a set of representatives of the cosets of H in K. Show that the products $\{s_j \cdot r_i\}$ form a set of representatives for the cosets of H in G. (In other words: Show that for every $x \in G$ there is a pair r, s such that $Hx = (s_jr_i)$, and that $H(s_jr_i) = H(s_yr_x)$ only if $i = x$ and $j = y$. **Hint:** It is easiest to first consider cosets of K to take care of the r-part.)
b) Show that $[G{:}H] = [G{:}K]\cdot[K{:}H]$. (You cannot use Lagrange's theorem, as we don't assume that G is finite!)

A first look at poster topics

The following is a rough list of suggestions for poster topics that I think would be suitable for this class. To help you with making a decision on the topic of your poster, you might want to start looking

at some of them (e.g. look at the topics in Wikipedia) already now, as they are of different level of formality or abstraction. (Obviously expectations on the poster are to be commensurable with the difficulty of the topic.) Topics with an (R) require ring theory, which we will cover later in the semester. I will give further guidance as the semester goes on, in particular if you chose any of these topics I will be able to give you some written material describing the topic. Topics are not exclusive. (If several of you want to do an **individual** poster on the same topic that is fine, but please try to not all pick the same few topics.) Your poster also should not be the same as you produce(d) for another class.

1. Block systems, Imprimitivity
2. Jordan-Hoelder theorem
3. Finite Simple Groups
4. Presentations
5. Semidirect Products
6. Nilpotent groups
7. Reflection groups
8. Frieze groups and/or Crystallographic groups (in dimensions 2, 3,…)
9. Fundamental groups
10. Fundamental theorem of finite abelian groups
11. Polya theory (counting with symmetries)
12. Group representations and characters
13. Group extensions
14. Geometric Group theory
15. Lie Groups
16. Automatic groups (if you know what an automaton is in CS)
17. Schreier-Sims algorithm / stabilizer chains
18. Double Cosets
19. Cyclotomic Polynomials (R)
20. Classification of finite fields (R)
21. Constructions with circle and ruler (R)
22. Invariant theory (R)
23. Algebraic varieties, Nullstellensatz (R)
24. Groebner bases (R)
25. Local Rings (R)
26. Algebraic integers (R)
27. Fractional Ideals (R)

Also, if you want to propose another topic (or application) relevant to the class, I am happy to get your suggestions. (Note that if you decide to make a poster about a person, the focus should be that person's mathematical work, not their life story.)