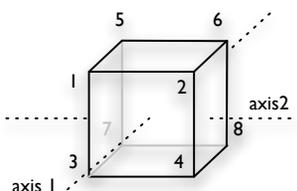


52) Consider the following permutations:

$$a = \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{matrix}, \quad b = \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 4 & 3 \end{matrix}$$

- a) Compute  $ab$ ,  $bab$  and  $a^I (= a^{-1})$ .
- b) Write  $a$  and  $b$  in cycle notation. Compute  $a^3$  in cycle notation.
- c) Let  $c = (1, 3, 5, 4, 6)$ . Compute  $ac$ .

53) Consider the following cube, whose corners we'll label with the numbers 1 to 8:



Write down permutations in cycle form that show how the corners change under the following operations

- a) Clockwise rotation by 90 degree around the axis (axis 1) that goes through the front and back side.
- b) Rotation (by 180 degree) around the axis (axis 2) going through the right and left side.

54) Which of the following sets are groups? Either show that the axioms are fulfilled or show which axiom is violated:

- a) Even integers under addition.
- b) Even integers under multiplication.
- c) Odd integers under addition.
- d)  $\{z \in \mathbb{C} \mid |z| = 1\}$  under ordinary multiplication.
- e)  $\{1, 2, 3\}$  under multiplication modulo 4.
- f)  $\{0, 1, 2, 3, 4\}$  under multiplication modulo 5.
- g) The set of all matrices in  $\mathbb{R}^{3 \times 3}$  with determinant  $-1$  (under matrix multiplication).
- h) The set of all matrices in  $\mathbb{R}^{3 \times 3}$  of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \quad a, b, c \in \mathbb{R}$$

under matrix multiplication.

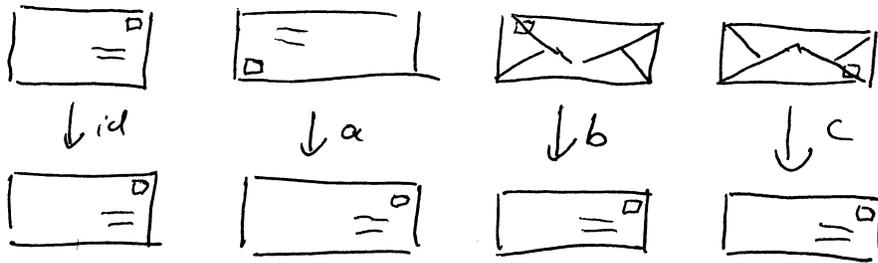
- i) For a given real number  $c$ , the numbers in  $(-c, c)$  with operation (this is the addition of velocities in special relativity)

$$x \circ y = \frac{x + y}{1 + xy/c^2}$$

**Hint:** You do not have to prove again statements we have proven more generally in the lecture (for example: Addition and Multiplication of complex numbers is associative, matrix multiplication is associative), but can just refer to it.

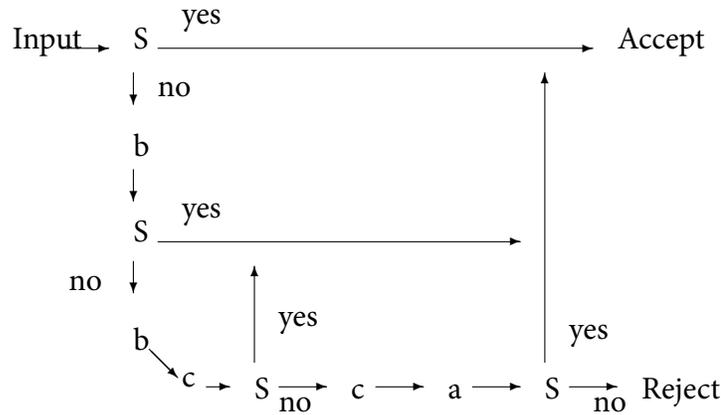
Do not forget to check that the set is closed under the operation and that inverse elements are indeed in the set!

55\*) The Irish postal service (this is a true story) used a machine (“automatic letter facer”) to turn letters in the right position for checking and validating the stamp. For this, it uses a mechanism that can (for example by “tipping the letters over”) perform the following transformations:



The machine then uses a stamp detector (S) that indicates whether a valid stamp was found in the upper right corner of the letter.

It applies the following checking process:



Apparently the people who built the machine did not realize that  $\{id, a, b, c\}$  form a group under composition. This makes it possible to simplify the construction:

- Compute the multiplication table for  $\{id, a, b, c\}$ .
- Suggests a simplified construction that does fewer letter flips (so processing is faster)
- The mechanism for the “rotation” a) is relatively complicated (and thus error prone). Suggest a simplified construction in which no operation of type a) is needed (thus simplifying the engineering).

Problems marked with a \* are bonus problems for extra credit.