

29] We know $\text{Order Mod}_3(a) = 30 = p-1$ and
 (prob. 19): $\text{Order Mod}_p(a^e) = \frac{30}{\gcd(30, e)}$. Thus

e	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\gcd(30, e)$	1	2	3	2	5	6	1	2	3	10	1	6	1	2	15
$\frac{30}{\gcd(30, e)}$	30	15	10	15	6	5	30	15	10	3	30	5	30	15	2
$30-e$	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15

Note that $\gcd(30, e) = \gcd(30-e, e)$, so the bottom row gives rest of table.

Order	2	3	5	6	10	15	30
Frequency	1	2	4	2	4	8	8

30] Rank of $\text{Matrix} = 3 = \dim V$
modulo 2

(columns 3, 4, 5 are independent, col 1 = col 4 + col 5
 col 2 = col 3 + col 4)

31) No, as 000000 is mapped to $1110 \neq 0000$

32) If you knew a basis of messages and their encryptions, you could encrypt/decrypt any message by linearity.

33) arithmetic

34) See Knuth, The art of computer programming
Vol. 2, problem 3.1