1) $1253 \bmod 24 = 5$, so its 3pm+5
= 8pm

2) We have $1 = 3\cdot 5 - 2\cdot 7 = 10\cdot 5 - 7\cdot 7$
so its not unique. As $1 = 5s + 7t$, $\gcd(s,t) = 1$

3b) Solve $87654320 + x \equiv 1 \pmod 7$
$\equiv 5 + x \Rightarrow x \equiv 1 - 5 \equiv -4 \equiv 3 \pmod 7$

3c) 87656220 and
87656227 both are valid and
differ only in the last digit, this could not
be recovered uniquely if damaged.

4) By the pigeonhole principle there exist
indices $m < n$ s.t. $3^m \equiv 3^n \pmod{10000}$.
Multiply with a^m, where a is the multiplicative
inverse of 3 mod 10000.

Then

$$1 \equiv a^m \cdot 3^m \equiv a^m \cdot 3^n \equiv 3^{\underbrace{n-m}_{>0 \text{ as } n>m}} \pmod{10000}$$

$\uparrow$ a is inverse of 3

and thus 3^{n-m} has last digit 0001.

5) is straightforward algebra.