Practice
§2.1: 13,16,20,29,31,33 §2.2: 3,5,9,12

Hand In
6) Find a general solution for the differential equations:
   a) $y' + \sin(t)y = 0$
   b) $y' + 2y = -t + e^{4t}$

7) Solve the initial value problem:
   $$y' + 5y = \sin(t), \quad y(0) = 1$$

8) Determine the general solution of the differential equation
   $$2\frac{dy}{dt} = y^2 \cdot \sin^2(t)$$

9) Consider the differential equation:
   $$\frac{dy}{dx} = \frac{x + x^3}{y}.$$
   a) Solve the initial value problem for $y(\frac{1}{2}) = 1$
   b) Solve the initial value problem for $y(\frac{1}{2}) = -1$
   c) For which $x$ values does the initial value problem $y(2) = 2$ have a solution?

10) Solve the Bernoulli equation
    $$\frac{dy}{dt} = 2y - 4y^2$$
11) (Problems 35,36 from §2.1:)

Variation of Parameters. Consider the following method of solving the general linear equation of first order:

\[ y' + p(t)y = g(t). \]  

(i)

(a) If \( g(t) \) is identically zero, show that the solution is

\[ y = A \exp \left[ - \int p(t) \, dt \right], \]

(ii)

where \( A \) is a constant.

(b) If \( g(t) \) is not identically zero, assume that the solution is of the form

\[ y = A(t) \exp \left[ - \int p(t) \, dt \right], \]

(iii)

where \( A \) is now a function of \( t \). By substituting for \( y \) in the given differential equation, show that \( A(t) \) must satisfy the condition

\[ A'(t) = g(t) \exp \left[ \int p(t) \, dt \right]. \]

(iv)

(c) Find \( A(t) \) from Eq. (iv). Then substitute for \( A(t) \) in Eq. (iii) and determine \( y \). This technique is known as the method of variation of parameters; it is discussed in detail in Section 3.7 in connection with second order linear equations.

Solve the differential equation \( y' - 2y = t^2e^{2t} \) using this method.