Practice

§9.1: 5, 9, 13
§9.2: 5, 7, 14
§9.3: 7, 9, 11
§9.5: 1, 5, 11

Hand In

(You may use MAPLE (or similar) to compute eigenvalues and eigenvectors. However you may not use the MatrixExponential function (or similar) to compute \( \exp(At) \) without any further work.

59\textsuperscript{*}) Solve the following initial value problem:

\[
\frac{d^2 x}{dt^2} = 4 \frac{dx}{dt} - x(t) + 6y(t), \quad x(0) = 1, x'(0) = 0
\]
\[
\frac{dy}{dt} = -x(t), \quad y(0) = 2
\]

60\textsuperscript{*}) Let

\[
A = \begin{pmatrix}
18 & 3 & 38 \\
177 & 41 & 473 \\
-19 & -4 & -47
\end{pmatrix} = M \cdot \begin{pmatrix}
4 & 1 & 0 \\
0 & 4 & 1 \\
0 & 0 & 4
\end{pmatrix} \cdot M^{-1} \quad \text{with} \quad M = \begin{pmatrix}
1 & 2 & -2 \\
8 & 29 & -28 \\
-1 & -3 & 3
\end{pmatrix}
\]

a) Calculate \( \exp(Jt) \) and \( \exp(At) \)

b) Determine a fundamental set of solutions for the system of differential equations \( \mathbf{x}' = A \cdot \mathbf{x} \).

c) Determine one particular solution for the system of differential equations

\[
\mathbf{x}' = A \cdot \mathbf{x} + \begin{pmatrix}
20\cos^2(t) + 12 & -32t \\
160\cos^2(t) + 174 & -448t \\
-20\cos^2(t) - 18 & 48t
\end{pmatrix}
\]

61\textsuperscript{*}) Classify the singularity at \((0,0)\) for the system

\[
\mathbf{x}' = \begin{pmatrix}
1 \\
3
\end{pmatrix} \cdot \mathbf{x}
\]

depending on the (real) value of \(a\).

**Hint:** First classify for which values of \(a\) the characteristic polynomial has multiple, and real or complex roots. Then determine the respective values of the roots.
Consider the following system of differential equations:

\[
\frac{dx}{dt} = (2+x)(y-x), \quad \frac{dy}{dt} = y(2+x+x^2)
\]

a) determine the critical points
b) (MAPLE) sketch the direction field
c) Determine which critical points are asymptotically stable or unstable, and classify their types.

Consider the following system of differential equations:

\[
\begin{align*}
 x' &= -10x + 10y \\
 y' &= 28x - y - xz \\
 z' &= xy - 8z/3
\end{align*}
\]

a) Determine the critical points of this system (there are 3).
b) Calculate the Jacobi matrix.
c) Consider the linearized problems at the critical points. Determine their stability.

Problems marked with a * are bonus problems for extra credit.