Mathematics 340-3  
Homework (due Nov. 11)  
A. Hulpke

Practice

§7.1: 1,3,4,5,6,15  
§7.2: 1,2,21,23,26

Hand In

46) Suppose you have three water containers, each containing a volume $V$ of water, connected as in the picture. Water, containing salt at a concentration $c_0$ is flowing into container $A$ at a rate of $r_0$ per minute. Furthermore we have the following flow rates between containers:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$B$</td>
<td>$r_0 + r_1$</td>
</tr>
<tr>
<td>$B$</td>
<td>$A$</td>
<td>$r_1$</td>
</tr>
<tr>
<td>$B$</td>
<td>$C$</td>
<td>$r_0 + r_2$</td>
</tr>
<tr>
<td>$C$</td>
<td>$B$</td>
<td>$r_2$</td>
</tr>
<tr>
<td>$C$</td>
<td>out</td>
<td>$r_0$</td>
</tr>
</tbody>
</table>

Set up a system of differential equations for the amount of salt in the containers.

47) Let $y(t)$ be a function fulfilling a second order linear differential equation initial value problem:

$$y'' + b \cdot y' + c \cdot y = g(t), \quad y(t_0) = y_0, y'(t_0) = y'_0$$

We set $w(t) := y'(t)$. Write the above differential equation as a system of two first order differential equations for $y$ and $w$.

48) For which values of $\lambda$ is the following matrix invertible?

$$
\begin{pmatrix}
2 & -\lambda & 2 \\
1 & 1 & \lambda \\
-2 & 1 & -2
\end{pmatrix}
$$

49) We define the exponential function of a matrix $A$ (in the same way as for numbers) as

$$\exp(A) := \sum_{i=0}^{\infty} \frac{1}{i!}A^i.$$ 

Using this definition, calculate

$$\exp\left( \begin{pmatrix} 2t & 0 \\ 0 & -5t \end{pmatrix} \right) \quad \text{and} \quad \exp\left( \begin{pmatrix} t & 1 \\ 0 & t \end{pmatrix} \right)$$