Practice
§1.1:7,15,16
§1.2:3,6,11
§1.3: 2,3,4,9,10,12,17,20

Hand In

1) For the following problems, find the order of the differential equation and state whether it is linear or nonlinear:

\[(1 + y'^2) \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = e^t\]  
\[\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1\]  
\[\frac{dy}{dt} + ty^2 = 0\]  
\[\frac{d^2y}{dt^2} + \sin(t + y) = \sin(t)\]

2) Verify that the given functions are solutions of the differential equation:

a) \(ty' - y = t^2, \quad y = 3t + t^2\)

b) \(t^2y'' + 5ty' + 4y = 0, \quad t > 0, \quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2} \ln(t)\).

c) \(y' - 2ty = 1, \quad y = e^t \int_0^t e^{-s^2} \, ds + e^t\)

3) Determine the value \(r\) for which the following differential equation has a solution of the form \(y = e^{rt}\):

\[y'' + y' - 6y = 0\]

4) Determine a solution for the initial value problem:

\[\frac{dy}{dt} = 3y + 4, \quad y(0) = 10\]

5) A student has bought a coffee at Sweet Sinsations just a few minutes before the lecture starts (to which it taking place in a computer lab – no coffee can be brought). As the coffee is still very hot, the student would like to have it as cool as possible in 2 minutes and she deliberates whether she should put in the milk immediately. Fortunately
she took a course in differential equations.

**Assume:** There are $c$ units of coffee at initial temperature $90^\circ C$ and $m = \frac{1}{10} \cdot c$ units of milk at fridge temperature $10^\circ C$. The room temperature is $20^\circ C$.

For the purpose of this problem assume that both water and coffee have the same specific heat: The temperature after mixing is $\frac{c \cdot T_{\text{coff}} + m \cdot T_{\text{milk}}}{c + m}$. Also both coffee and coffee/milk mixture cool down with coefficient $0.1 \cdot \text{sec}^{-1}$. We also assume that the milk is in the fridge and does not heat up before mixing.

a) What is the temperature when the coffee cools down for $t$ seconds and then the milk is mixed in?

b) What is the temperature after $t$ seconds when the milk is mixed in immediately at the start?

c) Should one mix in the milk first, or later?