

6a) Suppose that $n!$ ends in k zeroes, i.e.

$$n! = m \cdot 10^k. \text{ Then } (n+1)! = (n+1) \cdot n!$$

$= (n+1) \cdot m \cdot 10^k$, thus ends in (at least) k zeroes.

b). As there will be at least one 2 for every 5 we just need to count the powers of 5 that divide.

For a given n , there are $\left\lfloor \frac{n}{5} \right\rfloor$ multiples of 5,
 $\left\lfloor \frac{n}{25} \right\rfloor$ multiples of 25 (that give an extra 5)

and so on. We thus want

$$(*) \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{25} \right\rfloor + \left\lfloor \frac{n}{125} \right\rfloor + \left\lfloor \frac{n}{625} \right\rfloor + \left\lfloor \frac{n}{3125} \right\rfloor \geq 2017$$

will see that next term is zero.
to solve, ignore roundly for the moment.

$$n \left(\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \frac{1}{3125} \right) \geq 2017$$

$$= \frac{1 - (\frac{1}{5})^5}{1 - \frac{1}{5}} \cdot \frac{1}{5} \quad (\text{geom. series})$$

$$= \frac{781}{3125}, \quad 2017 \cdot \frac{3125}{781} = 8070.58.$$

Ans at least 8071. Now calculate exactly,

namely down: (x) for $n=8071$ gives 2014.
so at least 872000 after 5: $n=8075$ gives 2016.

Thus $n=8080$.

7) Solve $220 - (1.0357)^{30} - B \cdot \frac{1.0357 - 1}{0.0357} = 0$
for $B \Rightarrow B = 12.0668$.

Total 30 $B = 362.04$.

8) If 7 cars had at most 3 students each
it would transport $7 \cdot 3 = 21$ students, not enough.

a) Numbers ~~not~~ containing 9: ~~9, 19, 29, ..., 999~~ 9^{10}
Thus $10^{10} - 9^{10}$ different numbers containing 9
(this is larger)

b) The question is: ~~For which n~~ $\frac{9^n}{10^n} < \frac{1}{2}$, i.e. $\left(\frac{9}{10}\right)^n < \frac{1}{2}$

Thus $n < \log_{0.9} \left(\frac{1}{2}\right)$, i.e. $n < 6.57$.

Thus the problem holds for $n \geq 7$.

10) label ke $(n-1)$ points different from 1

in all $(n-1)!$ possible ways.