

17) Suppose we have n green balls and n gold balls and want to select n balls overall. We could do so by selecting the green balls (assume there are $k \leq n$ of them) first, and then the gold. Show (explain in text with full sentences) how this yields the identity

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}.$$

18) a) How many ways are there to sit k people on a row of $n > 2k$ seats, so that there is at least one free seat between two persons? (Hint: Assume you are placing dummies in the empty seats. How many dummies are there?)

b) How many ways exist if you allow (but do not require) that two people may sit near each other *once* in the row?

19) Show, using the definition, that $n^5 + 3n + 17 = \mathcal{O}(n^5)$.

20) Order the following functions according to their growth rate:

$$n, \sqrt{n}, n \log^2 n, n \log n, n \log \log n, n \log n^2, 2/n, 2^n, n+2, n^2 \log n, n^3$$

21) Determine the number of permutations of $1, \dots, 9$ (such permutations are called *derangements*) in which no digit is in its correct position.

22) A credit card has a 16 digit number and a secret 4-digit PIN number that is not supposed to appear on the card. However it could happen (as it would be in the picture if the PIN was 2641) that the PIN is already printed as part of the credit card number, violating the rule. We want to see how often this happens in an easier example:



Given a 3-digit PIN containing different¹ digits, how many 8-digit numbers contain the PIN at least once as a sequence of 3 consecutive digits?

(Hint: Determine the number of potential card numbers that have a PIN in positions 1-3, 2-4 etc. and subtract those which have it multiple times.)

B) Give an example of two positive, nondecreasing, functions f, g such that *neither* $f(n) = \mathcal{O}(g(n))$, nor $g(n) = \mathcal{O}(f(n))$.

About the Midterm: Bring writing implements, you will be given paper. You may use a pocket calculator that is incapable of transmitting data; it may not store any user-defined information. You also may bring a handwritten single page, letter size with notes.

¹this makes it easier as no self-overlap must be considered