Mathematics 301	**Final (100 points)**	Due by 12/12/09, 9:30AM

Points (leave blank)						
1	2	3	4	5	6	$\sum$

Name:
(clearly, please)

This exam is my own work. Sources (apart from the textbooks and my lecture notes) are indicated.
Honor pledge: I have not given, received, or used any unauthorized assistance.

Signature

Notes

- You are permitted to use your notes and any publication (book, journal, web page). You are not permitted to consult persons.
  Results which are quoted from a publication (apart from the course textbook, homework or your lecture notes) must be indicated. You also may use a calculator/computer for arithmetic.
- However citations or computer use that render a problem trivial ("This is lemma 4.3.2 in the book xxx", " I typed the question text into Wolfram alpha and it gave this answer") are not sufficient.
- You can work on the problems in any order you like. All problems carry equal weight.
- Show your work! Argument steps and explanation for statements made are a crucial part of a solution. Partial credit will be given sparingly – rather complete one problem than start two only partially.
- If you need extra sheets, staple them to this exam. (You do not need to submit scrap paper.)
- Return by the end of the scheduled exam time (Tuesday, 12/12, 9:30am) (in my office, the exam room, or the math front office), of course you may submit it earlier.
- I aim to put exam scores and grades on Canvas by the end of exam week. The formula for percentages is

  $$\frac{1}{4}\left(\frac{100}{61}\text{HWPOINTS} + \frac{100}{83}\text{MT1} + \text{MagnusBonus} + \text{MT2} + \text{FINAL}\right)$$

  with grades given as 90-100:A, 80-89:B, 70-79:C 60-69: D, below 60: F.

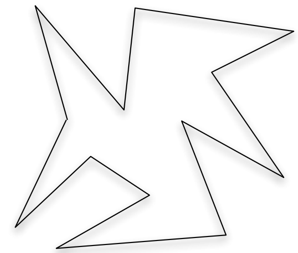
  I will not be able to give exam/grade information by email or phone and will not entertain email discussions about the grade obtained or hypothetical grades under better performance.
- Exams can be picked up after Christmas in the mathematics front office.

1) A florist has 5 aspidistras, 6 buttercups and 7 chrysanthemums. How many different kinds of bouquets of a dozen flowers (it is not required to use all types of flowers) can she make from these?

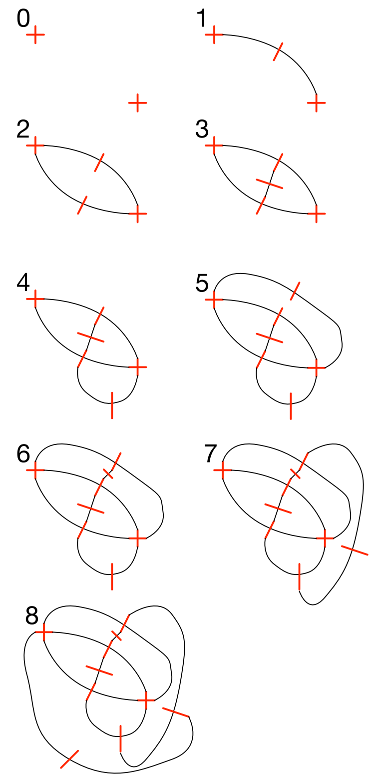
2) Give a *combinatorial proof* (no points will be given for a purely algebraic or induction proof) of the identity

$$\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}.$$

3) Consider a museum gallery (considered as 2-dimensional according to floor plan) shaped like a polygon with n vertices (as in the picture). Using the result from problem 59, show that it is possible to place $\lfloor \frac{n}{3} \rfloor$ guards in the room so that every point will be seen by a guard.



4) Two players play a game (invented by J.H. Conway and M.S. Paterson and called *Brussels Sprouts*) with a pen on a piece of paper. At the start n crosses will be placed randomly on the paper. Now players move alternatingly.
Each move consists of connecting two unused ends (that might belong to the same cross) with a line without touching or crossing other lines. Then a short line is drawn crossing this new line (creating a cross with two open ends).
A player who cannot move loses. The image on the side shows a game starting with $n = 2$ crosses, at stage 8 player one loses, as she cannot make any move.



We want to show that a game starting with n crosses ends after exactly $5n-2$ moves (and thus conclude that the first player wins if and only if n is odd).
Hints: (Steps for the proof) Consider the crosses as vertices and the lines forming a planar graph. Show that after m moves there are $2m$ edges and $n+m$ vertices.
Then show that any face has at least one end of a cross inside, and that the game ends if every face has only one end of a cross inside.
Using the fact that the number of free ends remains constant, conclude that there must be $4m$ faces when the game ends.
Finally use Euler's theorem to deduce a relation between m and n.

5) A mob of meerkats (all of different heights) is in rectangular formation in their enclosure. The zookeeper rearranges the meerkats in each row of the rectangle in decreasing order of height. He then rearranges the meerkats in each column in decreasing order of height.Prove that it is not necessary to rearrange the rows again; that is, the rows remain in decreasing order of height.

6) Let G be a graph of degree n with degree sequence $d_1 \geq d_2 \geq d_3 \geq \cdots d_n$. Show that the chromatic number satisfies:

$$\chi(G) \leq 1 + \max_i \min(d_i, i - 1).$$

Hint: Assign colors to vertices in order of nonincreasing degrees such that no conflict arises. This will produce a valid coloring.