**Mathematics 192** **Homework (due Sep. 10)** A. Hulpke

**1)** Consider $n$ straight lines *in general position* (that is no two lines are parallel, no two lines are identical, no three lines intersect in the same point) in the plane and let $R(n)$ be the number of regions they cut the plane into.

a) Determine (e.g. by a sketch) $R(0)$, $R(1)$, $R(2)$, $R(3)$, $R(4)$.
b) Let $D(n) := R(n+1) - R(n)$. Calculate $D(0)$, $D(1)$, $D(2)$, $D(3)$. Do you see a pattern?
c) It is claimed that $D(n)$ is a linear function, i.e. $D(n) = a \cdot n + b$ for suitable $a$, $b$. What are $a$, $b$? Can you explain this pattern?
d) Calculus tells us that $R(n)$ should have the form $en^2 + fn + g$. Using the values for $R(n)$ for $n = 0, 1, 2$ write doen a system of equations for the variable $e$, $f$, $g$. Solve the system.
e) Check the formula you got in d) for $R(5)$.
f) If you wanted to prove the formula in d), how could you proceed. Can you describe a way that would prove that the statement is true for any $n$? (I am looking for 2-4 sentences text, but I am not expecting a full proof.)

**2)** We consider the polynomial $E(x) = x^2 - x + 41$. Algernon claims that $E(x)$ is prime for any integer $x$. Myfanwy does not believe this claim.

a) Write the statement of the claim in a more formal form with quantifiers.
b) Calculate $E(x)$ for some small values of $x$. Is the result prime? Does this constitute a proof of Algernon's claim?
c) What would Myfanwy have to do to disproof the claim? What would you try if you only wanted to spend a few minutes on trying?
d) Is Algernon's statement true? Can you prove or disprove the claim?

**3)** Longs Peak is 14,259 feet high, Fort Collins is at roughly 5000 ft.

a) When walking up from Fort Collins to the summit of Longs Peak, will the tip of your right shoe be at exactly 10,000ft at any time? Explain.
b) A squirrel is running along this route and is never descending. Does this guarantee that it will reach the summit?
c) A band of crazy dwarves is building a stair up from Fort Collins to the summit. Will this stair have to include a step that is at exactly 10,000ft? Explain.

**4)** We want to find out (without using a calculator) whether $64^{65}$ or $65^{64}$ is larger.

a) Why is it sufficient to compare the logarithmized values $\log(64^{65})$, respectively $\log(64^{64})$? (It is most convenient to use the logarithm to base 2 here.)
b) Reduce the question to a comparison involving values $\log(64) = 6$ and $\log(65)$.
c) Can you answer the question? (E.g. using properties of the logarithm and the result from b).