Examples of arguments

You are about to leave for school in the morning and discover you don't have your glasses. You know the following statements are true:

a) If my glasses are on the kitchen table, then I saw them at breakfast.

b) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

c) If I was reading the newspaper in the living room, then my glasses are on the coffee table.

d) I did not see my glasses at breakfast.

e) If I was reading my book in bed, then my glasses are on the bed table.

f) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Where are the glasses?

Using

p My glasses are on the kitchen table.

q I saw my glasses at breakfast.

r I was reading the newspaper in the living room.

s I was reading the newspaper in the kitchen.

t My glasses are on the coffee table.

u I was reading my book in bed.

v My glasses are on the bed table.

we translate the statements as:

a) $p \to q$

b) $r \vee s$

c) $r \to t$

d) $\neg q$

e) $u \to v$

f) $s \to p$

and argue as follows:

$$\begin{array}{ll} & p \to q \\ & \neg q \\ \therefore & \neg p \end{array}$$

$$\begin{array}{ll} & s \to p \\ & \neg p \\ \therefore & \neg s \end{array}$$

$$\begin{array}{ll} & r \vee s \\ & \neg s \\ \therefore & r \end{array}$$

$$\begin{array}{ll} & r \to t \\ & r \\ \therefore & t \end{array}$$

The glasses are on the coffee table!
Source: Epp, Discrete Mathematics

Sherlock Holmes

> "And now we come to the great question as to the reason why. Robbery has not been the object of this murder, for nothing was taken. Was it politics, or was it [about a] a woman? That is the question confronting me. I was inclined from the first to the latter supposition. Political assassins are only too glad to do their work and fly. This murder had, on the contrary, been done most deliberately and the perpetrator had left his tracks all over the room, showing he had been there all the time." - A. Conan Doyle, *A Study in Scarlet*

We use the following propositions

p It was robbery.

q Something was taken.

r It was politics.

s It was a woman.

t The assassin left immediately.

u The assassin left tracks all over the room.

and translate Holmes' premises as

$$\begin{array}{l} p \to q \\ \neg q \\ \neg p \to r \vee s \\ r \to t \\ u \to \neg t \\ u \end{array}$$

and argue:

$$\begin{array}{ll} & p \to q \\ & \neg q \\ \therefore & \neg p \\ \\ & \neg p \to r \vee s \\ & \neg p \\ \therefore & r \vee s \\ \\ & u \to \neg t \\ & u \\ \therefore & \neg t \\ \\ & r \to t \\ & \neg t \\ \therefore & \neg r \\ \\ & r \vee s \\ & \neg r \\ \therefore & s \end{array}$$

We conclude that it was [about] a women

We could have written the argument shorter by not repeating statements. Lines starting with a ∴ are conclusions, lines without are premises (axioms):

$$\begin{array}{ll} & p \to q \\ & \neg q \\ \therefore & \neg p \\ & \neg p \to r \vee s \\ \therefore & r \vee s \\ & u \to \neg t \\ & u \\ \therefore & \neg t \\ & r \to t \\ \therefore & \neg r \\ \therefore & s \end{array}$$

Assumptions

The logician Raymond Smullyan describes in a puzzle an island containing two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who speak to you as follows:

> A says: B is a knight.
> B says: A and I are of opposite type.

What are A and B?

Suppose A is a knight.	
∴ What A says is true.	by definition of knight
∴ B is a knight also	That's what A said
∴ What B says is true.	by definition of knight
∴ A and B are of opposite types.	That's what B said
∴ Contradiction	A and B are both knights and are of opposite type.
∴ A is not a knight.	Negation of the assumption is true
∴ A is a knave.	Axiom: everyone is knight or knave
∴ What A says is false	Definition knave
∴ B is not a knight	Consequence of A's statement being false
∴ B is a knave also.	