

**23)** (Delayed from last week) Consider the sequence  $a_i = \frac{1-3 \cdot i^2}{5+i^2}$ . We claim that  $L = \lim_{i \rightarrow \infty} a_i = -3$ . Show (give a formal proof with  $N$  and  $\epsilon$ ) that it converges towards this limit.

**27)** You are given the following recursive algorithm, where `const` is a constant (ignore the fact that one could write a better algorithm for the same result):

```
myfun:=function(n)
  if n=0 then return 0;
  else if n=1 then return const+1;
  else return myfun(n-1)+myfun(n-2)+const;
  end if;
end function;
```

Let  $a_n$  be the number of additions required when evaluating `myfun(n)`. Give a recursive description for  $a_n$ .

**28)** We want to solve the equation  $x^5 + 3x - 17 = 0$ . For this, let  $f(x) = x^5 + 3x - 17$ , and note that  $f(0) < 0$  and  $f(2) > 0$ . We thus start with the interval from  $a = 0$  to  $b = 2$ . Now repeatedly calculate the middle point  $c = \frac{a+b}{2}$ , and depending on whether  $f(c) < 0$  replace  $a$  or  $b$  with  $c$  and consider the half size interval that must contain the solution. (That is, your first step will have  $c = 1$  and because  $f(1) < 0$  you replace  $a$  by  $c = 1$  and then next consider  $c = 1.5$ .) Repeat, until  $|b - a| \leq 0.001$ . (You are encouraged to have a computer program do the calculation.)

- What is the approximate solution?
- After how many iterations was this found?
- How many iterations would you predict (i.e. without trying it out) to get an error  $< 10^{-6}$ ?

**29)** A club has 100 members. A prediction of future membership for the next 15 years has the membership change (number of new members minus number of leavers) be given in the  $n$ -th year as  $n^2 - 12n + 20 = (n - 2)(n - 10)$ .

- In which years will the club reach maximal or minimal membership? What will it be?

You are explicitly forbidden to share course material with people outside the class, or with any websites that allow such access. This includes “homework help” sites or “test/homework data banks”.