

23) (Delayed one week) Consider the sequence $a_i = \frac{1-3 \cdot i^2}{5+i^2}$. We claim that $L = \lim_{i \rightarrow \infty} a_i = -3$. Show (give a formal proof with N and ϵ) that it converges towards this limit.

24) When we multiply two polynomials, the coefficients of the product are sums of products of the original coefficients. For example, we have

$$\begin{aligned} & (2x^2 + 3x + 4) \cdot (5x^2 + 6x + 7) \\ = & (2 \cdot 5)x^4 + (2 \cdot 6 + 3 \cdot 5)x^3 + (2 \cdot 7 + 3 \cdot 6 + 4 \cdot 5)x^2 + (3 \cdot 7 + 4 \cdot 6)x + (4 \cdot 7) \end{aligned}$$

(by multiplying out and collecting by powers of x .) The following problem is basically: “Write a program for polynomial multiplication”.

Your task: Determine a formula for the product $a(x)b(x)$ of two polynomials of degree n :

$$a(x) = \sum_{i=0}^n a_i x^i, \quad b(x) = \sum_{i=0}^n b_i x^i.$$

that reflects this “sum of products” form, as in the above example. That is, your answer will be of the form

$$a(x)b(x) = \sum_{i=0}^{\star} \left(\sum_{j=\star}^{\star} a_{\star} b_{\star} \right) x^i$$

where your task is to determine the bits marked by \star .

25) Write the repeating decimal $0.\overline{128205}$ as an exact fraction of two integers.

26) You are taking out a mortgage (of amount S dollars) on a house. The (fixed) interest rate is 0.3% per month (i.e. 3.6% per year). Each month you are able to pay back \$1000 for principal and interest. (That is, at the end of month 1, your loan sum is $S \cdot 1.003 - 1000$. We ignore taxes, insurance, etc.

a) Write down a formula for the loan sum after k months. Use your knowledge of the geometric sum formula to resolve a sum over k terms into a fraction. (Your solution will involve the variables S and k).

b) You want the loan to be paid off after 30 years (360 months). What is the (maximal) mortgage amount S you could borrow to achieve this? How much will you have paid in total over that period?

c) Using the same loan amount as in b), you increase your monthly payments to \$1100 (i.e. by 10%). How long will it take to pay off this amount? How much will you have paid in total over that period?

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