19) We define the sequence f_n of Fibonacci numbers recursively by

$$f_0 = 0, \qquad f_1 = 1, \qquad f_{n+2} = f_{n+1} + f_n.$$

a) Calculate f_{10}.
b) Show that this recursion is satisfied by the formula

$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right).$$

That means, you must show that:

1. The formula gives the correct values for f_0 and f_1.
2. Plug $n+1$ and $n+2$ in the formula and evaluate f_{n+1} and f_{n+2}.
3. Show that (with the formulas for f_{n+1} and f_{n+2} you just got) $f_{n+2} = f_{n+1} + f_n$ holds for every n.

20) Consider the sequence $a_n = \frac{5n+2}{n+5}$.
a) Show that the sequence is monotonically increasing (that is, show that $a_{n+1} > a_n$ for every n).
b) Show that this sequence a_n is bounded from above.

21) Consider the following sequences. Using increasingly larger values of n, try to determine their limits, as $n \to \infty$. Based on these (and maybe more), can you make a guess on what the limit of a sequence given by a rational function in n should be?

a) $\dfrac{3n+2}{4n-17}$

b) $\dfrac{5n^2+7n-1}{8n^2+3n+1}$

c) $\dfrac{5n^3+7n-1}{8n^2+3n+1}$

d) $\dfrac{5n^2+7n-1}{8n^3+3n+1}$

e) $\dfrac{3n^4+5n^3+1}{5n^4+12n-7}$

22) Consider the sequence $a_i = \frac{2^{i+1}}{i+3}$. Its first values are

$$1, 8/5, 8/3, 32/7, 8, \ldots$$

We want to *reindex* this sequence so that it starts at the value for a_3, that is we want do define a new sequence $b_i = a_{i+2}$. Give a formula for the value of b_i, depending only on i. Using this formula, calculate b_{20}.