Instructions

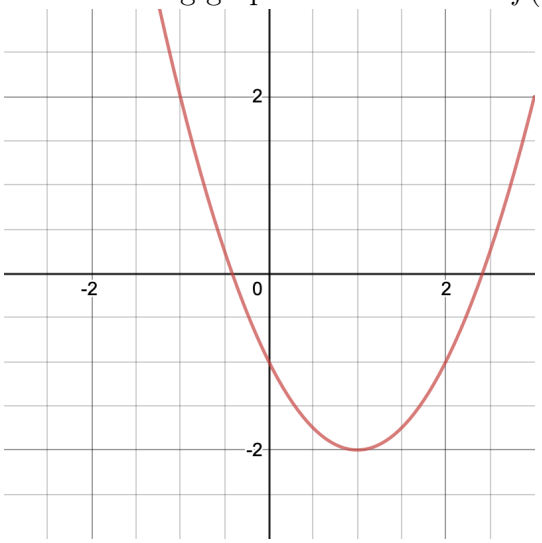
The weekly homework is supposed to be worked on paper (I recommend you work by hand on new sheets of paper and scan in the result) and the answer being uploaded.

14) Solve the following (systems of) equations modulo 7:
a) $(2 \cdot x + 3) \bmod 7 = 5$
b)

$$\begin{aligned} x - y &\mod 7 = 3 \\ 3x + 5y &\mod 7 = 2 \end{aligned}$$

c) Find (try out all possible values for x) the solutions for the equation $x^2 - 4x + 3 \bmod 7 = 0$.
d) Find the solutions for the equation $x^2 - 4x + 3 \bmod 8 = 0$. What is surprising about the result?

15) The following graph is of a function $f(x) = x^2 - 2x - 1$.



Based on this graph, sketch (in individual coordinate systems) the graphs for:
a) $f(x-1)$
b) $f(1-x)$
c) $1 - f(x)$
d) $f(1) - x$
e) $f(f(x))$

Make sure your graphs are clearly labelled!

15) Consider $A = \{1, 2, 3, 4, 5\}$ and the relation $p \subset A \times A$ given

$$\{(1,3), (2,2), (3,4), (4,1), (5,5)\}$$

a) Why is this relation a function?
b) Compute the compositions $q = p \circ p$ (for example by giving a list of pairs in $A \times A$), $r = q \circ p = (p \circ p) \circ p$, and $s = r \circ p$. (This process of composing again and again is called *iteration*.) Show that $p = s$.
c) Will this "return after iteration" work for every function? Can you find a function that will never return to itself when iterating?