Instructions

The weekly homework is supposed to be worked on paper (I recommend you work by hand on new sheets of paper and scan in the result) and the answer being uploaded. (You do not need to upload the problem sheet.) You may discuss the problems with other class participants, but the final write-up (not just copying an existing response) should be your own.

    Each problem carries the same weight and counts towards the homework points.

11)   Let $A = \{1, 2, \ldots, 20\}$. We define a relation P on A by

$$P = \{(a, b) \in A \times A \mid a \text{ and } b \text{ have the same set of prime factors}\}.$$

So for example 12P18 (as the prime factors are 2 and 3) but $4\cancel{P}6$. Show that it is an equivalence relation, and determine the equivalence classes (e.g. by giving each class as a list of elements).

12)   Consider the set of people with the relation "the first names share a letter". (That is *Maria* and *Robert* are in relation, but *Maria* and *Bob* are not.) Is this an equivalence relation? (Give a justification of your answer.)

13)   Consider the set $\mathbb{Q}$ of rational numbers. We define the *rounded value* of $x \in \mathbb{Q}$ as the integer round(x) closest to x (rounding up in the tie-break case that $x + \frac{1}{2}$ is an integer).

    We now define a relation R on $\mathbb{Q}$ by aRb if and only if round(a) = round(b).

    a) Show that this is an equivalence relation.

    b) What are the equivalence classes? (Some guiding questions: Can you describe the elements of the class of 1? How many classes are there? Can you describe (or sketch) the set of all classes?)

    c) Is this relation R the same as the "small difference" relation

$$\left\{ (a, b) \in \mathbb{Q} \times \mathbb{Q} \;\middle|\; |a - b| < \frac{1}{2} \right\} \quad ?$$

Explain your answer.