43) Determine the following indefinite integrals:

a) $\int 3x^5 - x^2 + 4x - 1 \, \mathrm{d}x$

b) $\int 3/x^2 \, \mathrm{d}x$

c) $\int 3\sin(3x + 2) \, \mathrm{d}x$

d) $\int \sum_{i=0}^{\infty} \frac{2}{i!}(x-2)^i \, \mathrm{d}x$ (Find an antiderivative using a Taylor series)

44) Solve the following indefinite integrals by the indicated substitution

a) $\int \frac{3x^2}{(x^3+2)^2} \, \mathrm{d}x$, Substitute $y = x^3 + 2$.

b) $\int (1 + \sin(x/5))^4 \cos(x/5) \, \mathrm{d}x$, Substitute $y = 1 + \sin(x/5)$.

c) $\int \frac{(3+\sqrt{x}))^{\frac{1}{4}}}{\sqrt{x}} \, \mathrm{d}x$, Substitute $y = 3 + \sqrt{x}$.

45) Use the reverse of the product rule (integration by parts) to solve the following integrals:

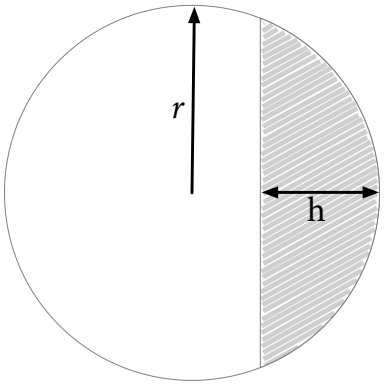
a) $\int x \cdot \cos(x) \mathrm{d}x$, Use $f(x) = x$ and $g'(x) = \cos(x)$.

b) $\int x \cdot \log(x) \mathrm{d}x$, Use $f(x) = \log(x)$ and $g'(x) = x$.

c) $\int x^2 \cdot \sin(x) \mathrm{d}x$, Use $f(x) = x^2$ and $g'(x) = \sin(x)$. You will then have to do a second (similar) integration by parts.

d) $\int \log(x) \mathrm{d}x$, Use $f(x) = \log(x)$ and $g'(x) = 1$ (i.e. you introduce an "invisible" factor of 1 and write $\log(x) = \log(x) \cdot 1$).

46) We cut a piece of height h from a circle with radius r. Determine a formula for the sliced off (shaded) area.



47) This is an example of a more complicated substitution situation. We want to integrate $\int \frac{1}{x\sqrt{x^2-1}}\mathrm{d}x$ by substituting $u = \sqrt{x^2-1}$. However there is the extra factor in the denominator, and no obvious factor for the inner derivative:

a) Set $u = \sqrt{x^2-1}$ and solve for x^2 in terms of u (we will need it later).

b) Calculate $B = \frac{\mathrm{d}u}{\mathrm{d}x}$.

c) As B does not occur in the integrand, multiply both numerator and denominator with x so that it occurs: $\int \frac{x}{x^2\sqrt{x^2-1}}\,\mathrm{d}x$. Now substitute $u = \sqrt{x^2-1}$: Replace $B\,\mathrm{d}x$ by $\mathrm{d}u$ and express the remaining x -bits in terms of u (using the result of a).

d) Solve the resulting integral in u (you will need the function arctan from homework problem 42) and substitute back.

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