Instructions

Please work these problems on a separate sheet. The solutions must include some kind of justification (such as the steps how to obtain it, or explanations).

37) Show that $x\log(x) = \mathcal{O}(x^4)$.

38) a) Compute the Taylor polynomial for $\cos(x)$ of degree 6, centered at $a = 0$.
b) Compute the Taylor polynomial for $\log(x)$ (natural logarithm) of degree 4, centered at $a = 1$.
c) Compute the Taylor polynomial for $\exp(x)$ of degree 3, centered at $a = 1$. Expand the terms $(x-1)^i$. Compare with the Taylor polynomial centered at $a = 0$.

39) a) Compute the error estimate for the Taylor polynomial for $\cos(x)$ of degree 6, centered at $a = 0$, when $-1 \le x \le 1$. (Use as estimate that $|\sin(x)|, |\cos(x)| \le M = 1$).
b) Compute the error estimate for the Taylor polynomial for $\log(x)$ (natural logarithm) of degree 4, centered at $a = 1$ when $1/2 \le x \le 3/2$. You may use that $\log(x)$ and all its derivatives are bounded by $M = 1$ on this interval.

40) Determine the Taylor series (around 0) for the following functions. (Use and modify Taylor series you know already, rather than computing them new from scratch):
a) $x^2 \cdot \exp(x^3)$.
b) $\sin(x) + \cos(x)$
c) $x^5 + 3x + 17$
d) $\frac{1}{3+x}$ (**Hint**: Write $x = -3y$ and consider a series in y)

41) A function (called arctan, it is the inverse of tangent) is given by the power series (=Taylor series)

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)}.$$

a) Determine the Taylor series for $f'(x)$.
b) Using geometric series, find a formula (not involving an infinite sum) for $f'(x)$.
Hint: Consider $-x^2$ as your variable q