4.5 EXERCISES

194. If c is a critical point of $f(x)$, when is there no local maximum or minimum at c? Explain.

195. For the function $y = x^3$, is $x = 0$ both an inflection point and a local maximum/minimum?

196. For the function $y = x^3$, is $x = 0$ an inflection point?

197. Is it possible for a point c to be both an inflection point and a local extrema of a twice differentiable function?

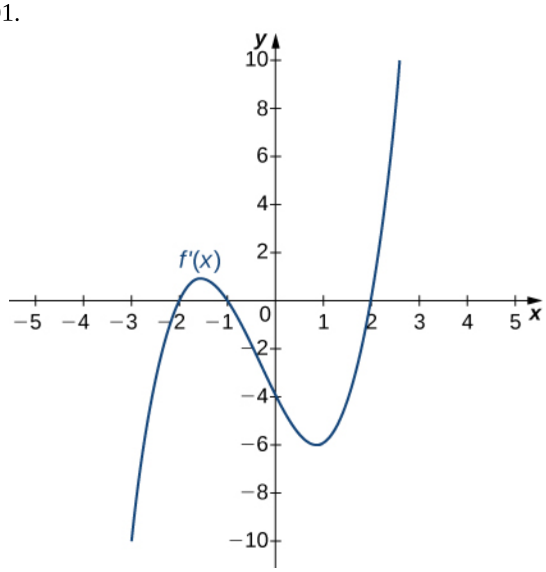
198. Why do you need continuity for the first derivative test? Come up with an example.

199. Explain whether a concave-down function has to cross $y = 0$ for some value of x.

200. Explain whether a polynomial of degree 2 can have an inflection point.

For the following exercises, analyze the graphs of f', then list all intervals where f is increasing or decreasing.

201.



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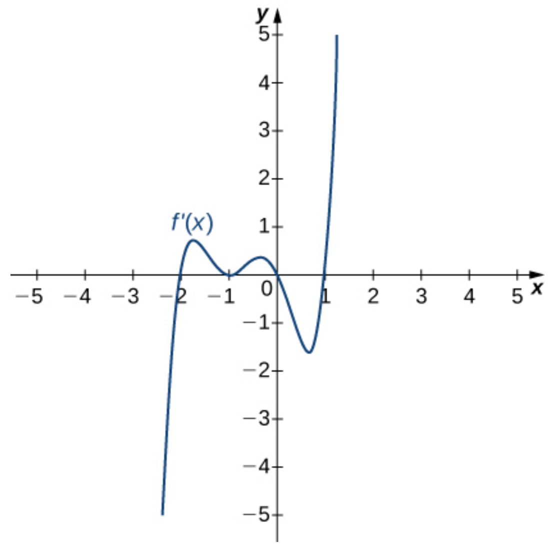

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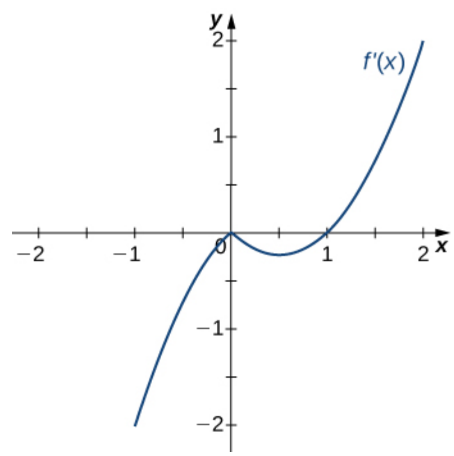

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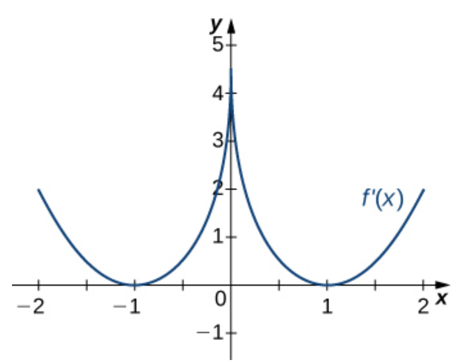

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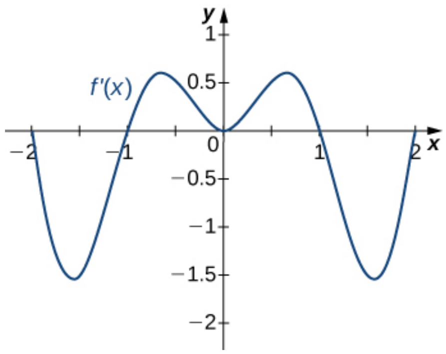
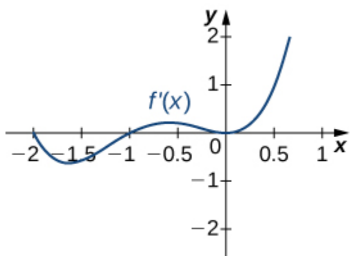


For the following exercises, analyze the graphs of f', then list all intervals where

a. f is increasing and decreasing and

b. the minima and maxima are located.

206.



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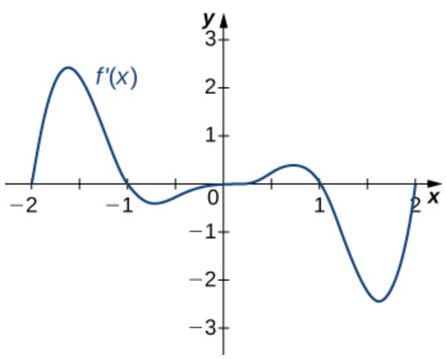

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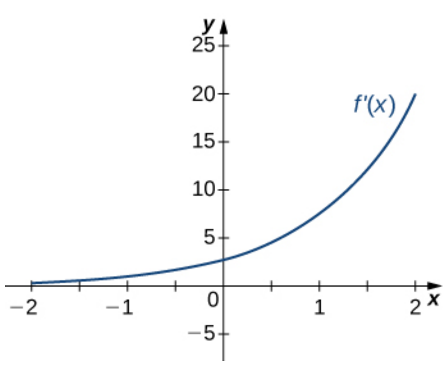

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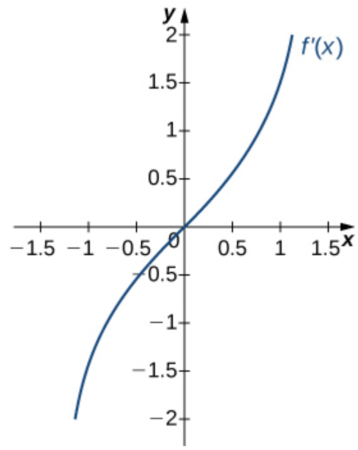

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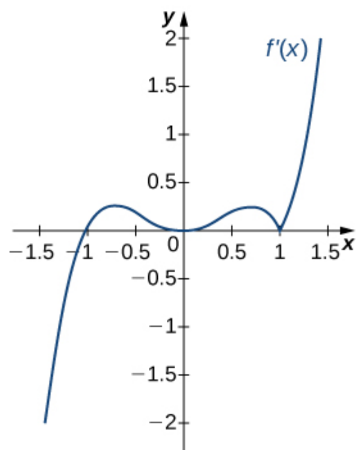
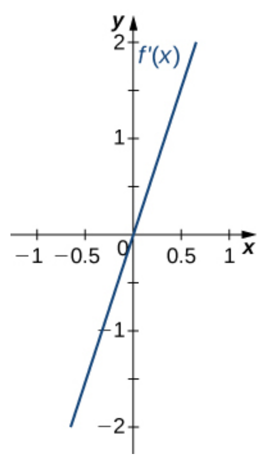


For the following exercises, analyze the graphs of f', then list all inflection points and intervals f that are concave up and concave down.

211.



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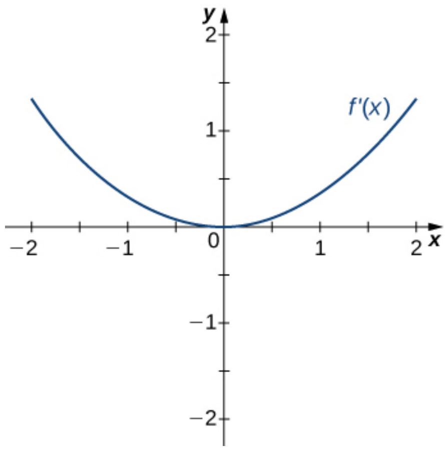

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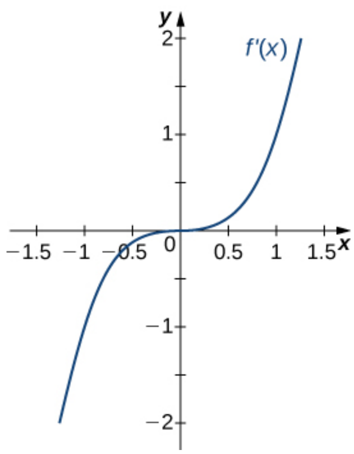

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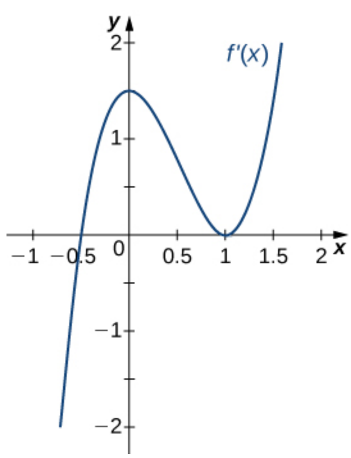

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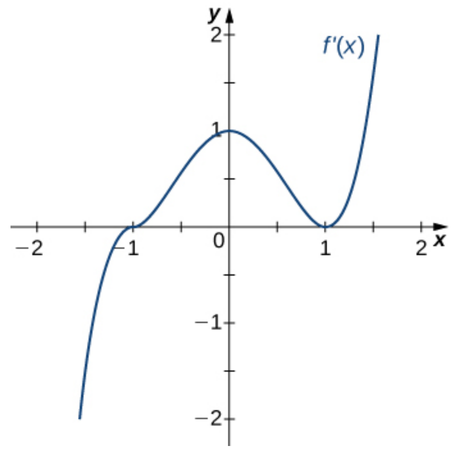


For the following exercises, draw a graph that satisfies the given specifications for the domain $x = [-3, 3]$. The function does not have to be continuous or differentiable.

216. $f(x) > 0,\ f'(x) > 0$ over $x > 1,\ -3 < x < 0,\ f'(x) = 0$ over $0 < x < 1$

217. $f'(x) > 0$ over $x > 2,\ -3 < x < -1,\ f'(x) < 0$ over $-1 < x < 2,\ f''(x) < 0$ for all x

218. $f''(x) < 0$ over $-1 < x < 1,\ f''(x) > 0,\ -3 < x < -1,\ 1 < x < 3,$ local maximum at $x = 0,$ local minima at $x = \pm 2$

219. There is a local maximum at $x = 2,$ local minimum at $x = 1,$ and the graph is neither concave up nor concave down.

220. There are local maxima at $x = \pm 1,$ the function is concave up for all $x,$ and the function remains positive for all $x.$

For the following exercises, determine

a. intervals where f is increasing or decreasing and

b. local minima and maxima of f.

221. $f(x) = \sin x + \sin^3 x$ over $-\pi < x < \pi$

222. $f(x) = x^2 + \cos x$

For the following exercises, determine a. intervals where f is concave up or concave down, and b. the inflection points of f.

223. $f(x) = x^3 - 4x^2 + x + 2$

For the following exercises, determine

a. intervals where f is increasing or decreasing,

b. local minima and maxima of f,

c. intervals where f is concave up and concave down, and

d. the inflection points of f.

224. $f(x) = x^2 - 6x$

225. $f(x) = x^3 - 6x^2$

226. $f(x) = x^4 - 6x^3$

227. $f(x) = x^{11} - 6x^{10}$

228. $f(x) = x + x^2 - x^3$

229. $f(x) = x^2 + x + 1$

230. $f(x) = x^3 + x^4$

For the following exercises, determine

a. intervals where f is increasing or decreasing,

b. local minima and maxima of f,

c. intervals where f is concave up and concave down, and

d. the inflection points of f. Sketch the curve, then use a calculator to compare your answer. If you cannot determine the exact answer analytically, use a calculator.

231. **[T]** $f(x) = \sin(\pi x) - \cos(\pi x)$ over $x = [-1, 1]$

232. **[T]** $f(x) = x + \sin(2x)$ over $x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

233. **[T]** $f(x) = \sin x + \tan x$ over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

234. **[T]** $f(x) = (x-2)^2(x-4)^2$

235. **[T]** $f(x) = \frac{1}{1-x}, \ x \neq 1$

236. **[T]** $f(x) = \frac{\sin x}{x}$ over $x = \ [2\pi, 0) \cup (0, 2\pi]$

237. $f(x) = \sin(x)e^x$ over $x = [-\pi, \pi]$

238. $f(x) = \ln x\sqrt{x}, \ x > 0$

239. $f(x) = \frac{1}{4}\sqrt{x} + \frac{1}{x}, \ x > 0$

240. $f(x) = \frac{e^x}{x}, \ x \neq 0$

For the following exercises, interpret the sentences in terms of f, f', and f''.

241. The population is growing more slowly. Here f is the population.

242. A bike accelerates faster, but a car goes faster. Here $f =$ Bike's position minus Car's position.

243. The airplane lands smoothly. Here f is the plane's altitude.

244. Stock prices are at their peak. Here f is the stock price.

245. The economy is picking up speed. Here f is a measure of the economy, such as GDP.

For the following exercises, consider a third-degree polynomial $f(x)$, which has the properties $f'(1) = 0, \ f'(3) = 0$. Determine whether the following statements are *true or false*. Justify your answer.

246. $f(x) = 0$ for some $1 \le x \le 3$

247. $f''(x) = 0$ for some $1 \le x \le 3$

248. There is no absolute maximum at $x = 3$

249. If $f(x)$ has three roots, then it has 1 inflection point.

250. If $f(x)$ has one inflection point, then it has three real roots.

5.5 EXERCISES

254. Why is u-substitution referred to as *change of variable*?

255. 2. If $f = g \circ h$, when reversing the chain rule, $\frac{d}{dx}(g \circ h)(x) = g'(h(x))h'(x)$, should you take $u = g(x)$ or $u = h(x)$?

In the following exercises, verify each identity using differentiation. Then, using the indicated u-substitution, identify f such that the integral takes the form $\int f(u)du$.

256. $\int x\sqrt{x+1}dx = \frac{2}{15}(x+1)^{3/2}(3x-2) + C;\ u = x+1$

257. For $x > 1: \int \frac{x^2}{\sqrt{x-1}}dx = \frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right) + C;\ u = x-1$

258. $\int x\sqrt{4x^2+9}dx = \frac{1}{12}\left(4x^2+9\right)^{3/2} + C;\ u = 4x^2+9$

259. $\int \frac{x}{\sqrt{4x^2+9}}dx = \frac{1}{4}\sqrt{4x^2+9} + C;\ u = 4x^2+9$

260. $\int \frac{x}{(4x^2+9)^2}dx = -\frac{1}{8(4x^2+9)};\ u = 4x^2+9$

In the following exercises, find the antiderivative using the indicated substitution.

261. $\int (x+1)^4 dx;\ u = x+1$

262. $\int (x-1)^5 dx;\ u = x-1$

263. $\int (2x-3)^{-7} dx;\ u = 2x-3$

264. $\int (3x-2)^{-11} dx;\ u = 3x-2$

265. $\int \frac{x}{\sqrt{x^2+1}}dx;\ u = x^2+1$

266. $\int \frac{x}{\sqrt{1-x^2}}dx;\ u = 1-x^2$

267. $\int (x-1)\left(x^2-2x\right)^3 dx;\ u = x^2-2x$

268. $\int \left(x^2-2x\right)\left(x^3-3x^2\right)^2 dx;\ u = x^3-3x^2$

269. $\int \cos^3\theta d\theta;\ u = \sin\theta\ \ (Hint: \cos^2\theta = 1-\sin^2\theta)$

270. $\int \sin^3\theta d\theta;\ u = \cos\theta\ \ (Hint: \sin^2\theta = 1-\cos^2\theta)$

In the following exercises, use a suitable change of variables to determine the indefinite integral.

271. $\int x(1-x)^{99} dx$

272. $\int t\left(1-t^2\right)^{10} dt$

273. $\int (11x-7)^{-3} dx$

274. $\int (7x-11)^4 dx$

275. $\int \cos^3\theta \sin\theta d\theta$

276. $\int \sin^7\theta \cos\theta d\theta$

277. $\int \cos^2(\pi t)\sin(\pi t)dt$

278. $\int \sin^2 x\cos^3 x dx\ \ (Hint: \sin^2 x + \cos^2 x = 1)$

279. $\int t\sin\left(t^2\right)\cos\left(t^2\right)dt$

280. $\int t^2\cos^2\left(t^3\right)\sin\left(t^3\right)dt$

281. $\int \frac{x^2}{\left(x^3-3\right)^2}dx$

282. $\int \frac{x^3}{\sqrt{1-x^2}}dx$

283. $\displaystyle\int \frac{y^5}{\left(1 - y^3\right)^{3/2}} dy$

284. $\displaystyle\int \cos\theta(1 - \cos\theta)^{99} \sin\theta d\theta$

285. $\displaystyle\int \left(1 - \cos^3\theta\right)^{10} \cos^2\theta \sin\theta d\theta$

286. $\displaystyle\int (\cos\theta - 1)\left(\cos^2\theta - 2\cos\theta\right)^3 \sin\theta d\theta$

287. $\displaystyle\int \left(\sin^2\theta - 2\sin\theta\right)\left(\sin^3\theta - 3\sin^2\theta\right)^3 \cos\theta d\theta$

In the following exercises, use a calculator to estimate the area under the curve using left Riemann sums with 50 terms, then use substitution to solve for the exact answer.

288. **[T]** $y = 3(1 - x)^2$ over $[0, 2]$

289. **[T]** $y = x\left(1 - x^2\right)^3$ over $[-1, 2]$

290. **[T]** $y = \sin x(1 - \cos x)^2$ over $[0, \pi]$

291. **[T]** $y = \dfrac{x}{\left(x^2 + 1\right)^2}$ over $[-1, 1]$

In the following exercises, use a change of variables to evaluate the definite integral.

292. $\displaystyle\int_0^1 x\sqrt{1 - x^2} dx$

293. $\displaystyle\int_0^1 \frac{x}{\sqrt{1 + x^2}} dx$

294. $\displaystyle\int_0^2 \frac{t^2}{\sqrt{5 + t^2}} dt$

295. $\displaystyle\int_0^1 \frac{t^2}{\sqrt{1 + t^3}} dt$

296. $\displaystyle\int_0^{\pi/4} \sec^2\theta \tan\theta d\theta$

297. $\displaystyle\int_0^{\pi/4} \frac{\sin\theta}{\cos^4\theta} d\theta$

In the following exercises, evaluate the indefinite integral $\int f(x)dx$ with constant $C = 0$ using u-substitution. Then, graph the function and the antiderivative over the indicated interval. If possible, estimate a value of C that would need to be added to the antiderivative to make it equal to the definite integral $F(x) = \int_a^x f(t)dt,$ with a the left endpoint of the given interval.

298. **[T]** $\displaystyle\int (2x + 1)e^{x^2 + x - 6} dx$ over $[-3, 2]$

299. **[T]** $\displaystyle\int \frac{\cos(\ln(2x))}{x} dx$ on $[0, 2]$

300. **[T]** $\displaystyle\int \frac{3x^2 + 2x + 1}{\sqrt{x^3 + x^2 + x + 4}} dx$ over $[-1, 2]$

301. **[T]** $\displaystyle\int \frac{\sin x}{\cos^3 x} dx$ over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

302. **[T]** $\displaystyle\int (x + 2)e^{-x^2 - 4x + 3} dx$ over $[-5, 1]$

303. **[T]** $\displaystyle\int 3x^2\sqrt{2x^3 + 1} dx$ over $[0, 1]$

304. If $h(a) = h(b)$ in $\int_a^b g'(h(x))h(x)dx,$ what can you say about the value of the integral?

305. Is the substitution $u = 1 - x^2$ in the definite integral $\int_0^2 \frac{x}{1 - x^2} dx$ okay? If not, why not?

In the following exercises, use a change of variables to show that each definite integral is equal to zero.

306. $\displaystyle\int_0^{\pi} \cos^2(2\theta)\sin(2\theta) d\theta$

307. $\displaystyle\int_0^{\sqrt{\pi}} t\cos\left(t^2\right)\sin\left(t^2\right) dt$

308. $\displaystyle\int_0^1 (1 - 2t) dt$

309. $\displaystyle\int_0^1 \frac{1-2t}{\left(1+\left(t-\frac{1}{2}\right)^2\right)}dt$

310. $\displaystyle\int_0^{\pi} \sin\left(\left(t-\frac{\pi}{2}\right)^3\right)\cos\left(t-\frac{\pi}{2}\right)dt$

311. $\displaystyle\int_0^2 (1-t)\cos(\pi t)dt$

312. $\displaystyle\int_{\pi/4}^{3\pi/4} \sin^2 t\cos t dt$

313. Show that the average value of $f(x)$ over an interval $[a, b]$ is the same as the average value of $f(cx)$ over the interval $\left[\frac{a}{c}, \frac{b}{c}\right]$ for $c > 0$.

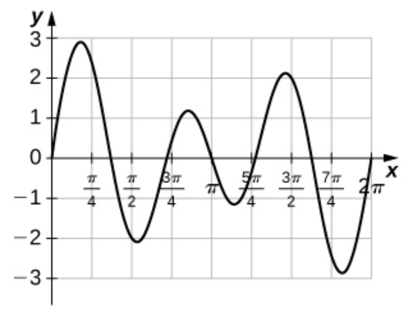
314. Find the area under the graph of $f(t) = \dfrac{t}{\left(1+t^2\right)^a}$ between $t = 0$ and $t = x$ where $a > 0$ and $a \neq 1$ is fixed, and evaluate the limit as $x \to \infty$.

315. Find the area under the graph of $g(t) = \dfrac{t}{\left(1-t^2\right)^a}$ between $t = 0$ and $t = x$, where $0 < x < 1$ and $a > 0$ is fixed. Evaluate the limit as $x \to 1$.

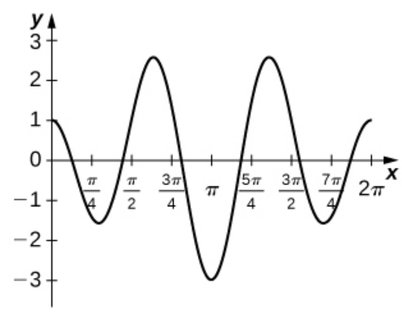
316. The area of a semicircle of radius 1 can be expressed as $\displaystyle\int_{-1}^{1}\sqrt{1-x^2}dx$. Use the substitution $x = \cos t$ to express the area of a semicircle as the integral of a trigonometric function. You do not need to compute the integral.

317. The area of the top half of an ellipse with a major axis that is the *x*-axis from $x = a$ to *a* and with a minor axis that is the *y*-axis from $y = -b$ to *b* can be written as $\displaystyle\int_{-a}^{a} b\sqrt{1-\frac{x^2}{a^2}}dx$. Use the substitution $x = a\cos t$ to express this area in terms of an integral of a trigonometric function. You do not need to compute the integral.

318. **[T]** The following graph is of a function of the form $f(t) = a\sin(nt) + b\sin(mt)$. Estimate the coefficients *a* and *b*, and the frequency parameters *n* and *m*. Use these estimates to approximate $\displaystyle\int_0^{\pi} f(t)dt$.



319. **[T]** The following graph is of a function of the form $f(x) = a\cos(nt) + b\cos(mt)$. Estimate the coefficients *a* and *b* and the frequency parameters *n* and *m*. Use these estimates to approximate $\displaystyle\int_0^{\pi} f(t)dt$.



5.6 EXERCISES

In the following exercises, compute each indefinite integral.

320. $\int e^{2x} dx$

321. $\int e^{-3x} dx$

322. $\int 2^x dx$

323. $\int 3^{-x} dx$

324. $\int \frac{1}{2x} dx$

325. $\int \frac{2}{x} dx$

326. $\int \frac{1}{x^2} dx$

327. $\int \frac{1}{\sqrt{x}} dx$

In the following exercises, find each indefinite integral by using appropriate substitutions.

328. $\int \frac{\ln x}{x} dx$

329. $\int \frac{dx}{x(\ln x)^2}$

330. $\int \frac{dx}{x \ln x} \ (x > 1)$

331. $\int \frac{dx}{x \ln x \ln(\ln x)}$

332. $\int \tan\theta \, d\theta$

333. $\int \frac{\cos x - x \sin x}{x \cos x} dx$

334. $\int \frac{\ln(\sin x)}{\tan x} dx$

335. $\int \ln(\cos x) \tan x dx$

336. $\int x e^{-x^2} dx$

337. $\int x^2 e^{-x^3} dx$

338. $\int e^{\sin x} \cos x dx$

339. $\int e^{\tan x} \sec^2 x dx$

340. $\int e^{\ln x} \frac{dx}{x}$

341. $\int \frac{e^{\ln(1-t)}}{1-t} dt$

In the following exercises, verify by differentiation that $\int \ln x \, dx = x(\ln x - 1) + C$, then use appropriate changes of variables to compute the integral.

342. $\int \ln x dx$ $\left(\textit{Hint}: \int \ln x dx = \frac{1}{2} \int x \ln\left(x^2\right) dx\right)$

343. $\int x^2 \ln^2 x \, dx$

344. $\int \frac{\ln x}{x^2} dx$ $\left(\textit{Hint}: \text{Set } u = \frac{1}{x}.\right)$

345. $\int \frac{\ln x}{\sqrt{x}} dx$ $\left(\textit{Hint}: \text{Set } u = \sqrt{x}.\right)$

346. Write an integral to express the area under the graph of $y = \frac{1}{t}$ from $t = 1$ to e^x and evaluate the integral.

347. Write an integral to express the area under the graph of $y = e^t$ between $t = 0$ and $t = \ln x$, and evaluate the integral.

In the following exercises, use appropriate substitutions to express the trigonometric integrals in terms of compositions with logarithms.

348. $\int \tan(2x) dx$

349. $\int \frac{\sin(3x) - \cos(3x)}{\sin(3x) + \cos(3x)} dx$

350. $\int \frac{x \sin\left(x^2\right)}{\cos\left(x^2\right)} dx$

351. $\int x \csc\left(x^2\right) dx$

352. $\int \ln(\cos x)\tan x\, dx$

353. $\int \ln(\csc x)\cot x dx$

354. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

In the following exercises, evaluate the definite integral.

355. $\int_1^2 \frac{1 + 2x + x^2}{3x + 3x^2 + x^3} dx$

356. $\int_0^{\pi/4} \tan x\, dx$

357. $\int_0^{\pi/3} \frac{\sin x - \cos x}{\sin x + \cos x} dx$

358. $\int_{\pi/6}^{\pi/2} \csc x dx$

359. $\int_{\pi/4}^{\pi/3} \cot x dx$

In the following exercises, integrate using the indicated substitution.

360. $\int \frac{x}{x - 100} dx;\ u = x - 100$

361. $\int \frac{y - 1}{y + 1} dy;\ u = y + 1$

362. $\int \frac{1 - x^2}{3x - x^3} dx;\ u = 3x - x^3$

363. $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx;\ u = \sin x - \cos x$

364. $\int e^{2x}\sqrt{1 - e^{2x}} dx;\ u = e^{2x}$

365. $\int \ln(x)\frac{\sqrt{1 - (\ln x)^2}}{x} dx;\ u = \ln x$

In the following exercises, does the right-endpoint approximation overestimate or underestimate the exact area? Calculate the right endpoint estimate R_{50} and solve for the exact area.

366. **[T]** $y = e^x$ over $[0, 1]$

367. **[T]** $y = e^{-x}$ over $[0, 1]$

368. **[T]** $y = \ln(x)$ over $[1, 2]$

369. **[T]** $y = \frac{x + 1}{x^2 + 2x + 6}$ over $[0, 1]$

370. **[T]** $y = 2^x$ over $[-1, 0]$

371. **[T]** $y = -2^{-x}$ over $[0, 1]$

In the following exercises, $f(x) \geq 0$ for $a \leq x \leq b$. Find the area under the graph of $f(x)$ between the given values a and b by integrating.

372. $f(x) = \frac{\log_{10}(x)}{x};\ a = 10,\ b = 100$

373. $f(x) = \frac{\log_2(x)}{x};\ a = 32,\ b = 64$

374. $f(x) = 2^{-x};\ a = 1,\ b = 2$

375. $f(x) = 2^{-x};\ a = 3,\ b = 4$

376. Find the area under the graph of the function $f(x) = xe^{-x^2}$ between $x = 0$ and $x = 5$.

377. Compute the integral of $f(x) = xe^{-x^2}$ and find the smallest value of N such that the area under the graph $f(x) = xe^{-x^2}$ between $x = N$ and $x = N + 1$ is, at most, 0.01.

378. Find the limit, as N tends to infinity, of the area under the graph of $f(x) = xe^{-x^2}$ between $x = 0$ and $x = 5$.

379. Show that $\int_a^b \frac{dt}{t} = \int_{1/b}^{1/a} \frac{dt}{t}$ when $0 < a \leq b$.

380. Suppose that $f(x) > 0$ for all x and that f and g are differentiable. Use the identity $f^g = e^{g \ln f}$ and the chain rule to find the derivative of f^g.

381. Use the previous exercise to find the antiderivative of $h(x) = x^x(1 + \ln x)$ and evaluate $\int_2^3 x^x(1 + \ln x)dx$.

382. Show that if $c > 0$, then the integral of $1/x$ from ac to bc $(0 < a < b)$ is the same as the integral of $1/x$ from a to b.

The following exercises are intended to derive the fundamental properties of the natural log starting from the *definition* $\ln(x) = \int_1^x \frac{dt}{t}$, using properties of the definite integral and making no further assumptions.

383. Use the identity $\ln(x) = \int_1^x \frac{dt}{t}$ to derive the identity $\ln\left(\frac{1}{x}\right) = -\ln x$.

384. Use a change of variable in the integral $\int_1^{xy} \frac{1}{t} dt$ to show that $\ln xy = \ln x + \ln y$ for $x, \ y > 0$.

385. Use the identity $\ln x = \int_1^x \frac{dt}{x}$ to show that $\ln(x)$ is an increasing function of x on $[0, \infty)$, and use the previous exercises to show that the range of $\ln(x)$ is $(-\infty, \infty)$. Without any further assumptions, conclude that $\ln(x)$ has an inverse function defined on $(-\infty, \infty)$.

386. Pretend, for the moment, that we do not know that e^x is the inverse function of $\ln(x)$, but keep in mind that $\ln(x)$ has an inverse function defined on $(-\infty, \infty)$. Call it E. Use the identity $\ln xy = \ln x + \ln y$ to deduce that $E(a + b) = E(a)E(b)$ for any real numbers a, b.

387. Pretend, for the moment, that we do not know that e^x is the inverse function of $\ln x$, but keep in mind that $\ln x$ has an inverse function defined on $(-\infty, \infty)$. Call it E. Show that $E'(t) = E(t)$.

388. The sine integral, defined as $S(x) = \int_0^x \frac{\sin t}{t} dt$ is an important quantity in engineering. Although it does not have a simple closed formula, it is possible to estimate its behavior for large x. Show that for $k \geq 1, \ |S(2\pi k) - S(2\pi(k + 1))| \leq \frac{1}{k(2k + 1)\pi}$.
$(Hint: \sin(t + \pi) = -\sin t)$

389. **[T]** The normal distribution in probability is given by $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x - \mu)^2/2\sigma^2}$, where σ is the standard deviation and μ is the average. The *standard normal distribution* in probability, p_s, corresponds to $\mu = 0$ and $\sigma = 1$. Compute the right endpoint estimates R_{10} and R_{100} of $\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-x^{2/2}} dx$.

390. **[T]** Compute the right endpoint estimates R_{50} and R_{100} of $\int_{-3}^{5} \frac{1}{2\sqrt{2\pi}} e^{-(x - 1)^2/8}$.

3.1 EXERCISES

In using the technique of integration by parts, you must carefully choose which expression is *u*. For each of the following problems, use the guidelines in this section to choose *u*. Do **not** evaluate the integrals.

1. $\int x^3 e^{2x} dx$

2. $\int x^3 \ln(x) dx$

3. $\int y^3 \cos y dx$

4. $\int x^2 \arctan x\, dx$

5. $\int e^{3x} \sin(2x) dx$

Find the integral by using the simplest method. Not all problems require integration by parts.

6. $\int v \sin v dv$

7. $\int \ln x\, dx$ (*Hint:* $\int \ln x\, dx$ is equivalent to $\int 1 \cdot \ln(x) dx$.)

8. $\int x \cos x\, dx$

9. $\int \tan^{-1} x\, dx$

10. $\int x^2 e^x dx$

11. $\int x \sin(2x) dx$

12. $\int x e^{4x} dx$

13. $\int x e^{-x} dx$

14. $\int x \cos 3x\, dx$

15. $\int x^2 \cos x\, dx$

16. $\int x \ln x\, dx$

17. $\int \ln(2x+1) dx$

18. $\int x^2 e^{4x} dx$

19. $\int e^x \sin x\, dx$

20. $\int e^x \cos x\, dx$

21. $\int x e^{-x^2} dx$

22. $\int x^2 e^{-x} dx$

23. $\int \sin(\ln(2x)) dx$

24. $\int cos(\ln x) dx$

25. $\int (\ln x)^2 dx$

26. $\int \ln(x^2) dx$

27. $\int x^2 \ln x\, dx$

28. $\int \sin^{-1} x\, dx$

29. $\int \cos^{-1}(2x) dx$

30. $\int x \arctan x\, dx$

31. $\int x^2 \sin x\, dx$

32. $\int x^3 \cos x\, dx$

33. $\int x^3 \sin x\, dx$

34. $\int x^3 e^x dx$

35. $\int x \sec^{-1} x\, dx$

36. $\int x \sec^2 x\, dx$

37. $\int x \cosh x\, dx$

Compute the definite integrals. Use a graphing utility to confirm your answers.

38. $\int_{1/e}^{1} \ln x\,dx$

39. $\int_{0}^{1} xe^{-2x}\,dx$ (Express the answer in exact form.)

40. $\int_{0}^{1} e^{\sqrt{x}}\,dx (\text{let } u = \sqrt{x})$

41. $\int_{1}^{e} \ln(x^2)dx$

42. $\int_{0}^{\pi} x\cos x\,dx$

43. $\int_{-\pi}^{\pi} x\sin x\,dx$ (Express the answer in exact form.)

44. $\int_{0}^{3} \ln(x^2+1)dx$ (Express the answer in exact form.)

45. $\int_{0}^{\pi/2} x^2 \sin x\,dx$ (Express the answer in exact form.)

46. $\int_{0}^{1} x5^x\,dx$ (Express the answer using five significant digits.)

47. Evaluate $\int \cos x \ln(\sin x)dx$

Derive the following formulas using the technique of integration by parts. Assume that n is a positive integer. These formulas are called *reduction formulas* because the exponent in the x term has been reduced by one in each case. The second integral is simpler than the original integral.

48. $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$

49. $\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1} \sin x\,dx$

50. $\int x^n \sin x\,dx =$ ______

51. Integrate $\int 2x\sqrt{2x-3}dx$ using two methods:

a. Using parts, letting $dv = \sqrt{2x-3}\,dx$
b. Substitution, letting $u = 2x-3$

State whether you would use integration by parts to evaluate the integral. If so, identify u and dv. If not, describe the technique used to perform the integration without actually doing the problem.

52. $\int x\ln x\,dx$

53. $\int \frac{\ln^2 x}{x}dx$

54. $\int xe^x\,dx$

55. $\int xe^{x^2-3}\,dx$

56. $\int x^2 \sin x\,dx$

57. $\int x^2 \sin(3x^3+2)dx$

Sketch the region bounded above by the curve, the x-axis, and $x = 1$, and find the area of the region. Provide the exact form or round answers to the number of places indicated.

58. $y = 2xe^{-x}$ (Approximate answer to four decimal places.)

59. $y = e^{-x}\sin(\pi x)$ (Approximate answer to five decimal places.)

Find the volume generated by rotating the region bounded by the given curves about the specified line. Express the answers in exact form or approximate to the number of decimal places indicated.

60. $y = \sin x,\ y = 0,\ x = 2\pi,\ x = 3\pi$ about the y-axis (Express the answer in exact form.)

61. $y = e^{-x}$ $y = 0,\ x = -1 x = 0;$ about $x = 1$ (Express the answer in exact form.)

62. A particle moving along a straight line has a velocity of $v(t) = t^2 e^{-t}$ after t sec. How far does it travel in the first 2 sec? (Assume the units are in feet and express the answer in exact form.)

63. Find the area under the graph of $y = \sec^3 x$ from $x = 0$ to $x = 1$. (Round the answer to two significant digits.)

64. Find the area between $y = (x-2)e^x$ and the x-axis from $x = 2$ to $x = 5$. (Express the answer in exact form.)

65. Find the area of the region enclosed by the curve $y = x\cos x$ and the x-axis for $\frac{11\pi}{2} \le x \le \frac{13\pi}{2}$. (Express the answer in exact form.)

66. Find the volume of the solid generated by revolving the region bounded by the curve $y = \ln x$, the x-axis, and the vertical line $x = e^2$ about the x-axis. (Express the answer in exact form.)

67. Find the volume of the solid generated by revolving the region bounded by the curve $y = 4\cos x$ and the x-axis, $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, about the x-axis. (Express the answer in exact form.)

68. Find the volume of the solid generated by revolving the region in the first quadrant bounded by $y = e^x$ and the x-axis, from $x = 0$ to $x = \ln(7)$, about the y-axis. (Express the answer in exact form.)