

2. diverges by the Direct Comparison Test since $n+n+n>n+\sqrt{n}+0 \ \Rightarrow \ \frac{3}{n+\sqrt{n}}>\frac{1}{n}$, which is the nth term of the divergent series $\sum_{n=1}^{\infty} \ \frac{1}{n}$ or use Limit Comparison Test with $b_n = \frac{1}{n}$

14. converges by the Limit Comparison Test (part 2) with $\frac{1}{n^{5/4}}$, the nth term of a convergent p-series:

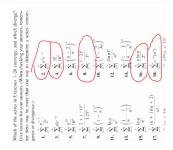
$$\lim_{n \to \infty} \frac{\lim_{n \to \infty^{+}} \left(\frac{\ln n^{2}}{n^{2}} \right)}{\left(\frac{2\pi n^{2}}{n^{2}} \right)} = \lim_{n \to \infty} \frac{(\ln n)^{2}}{n^{2}} = \lim_{n \to \infty} \frac{\left(\frac{2\ln n}{n} \right)}{\left(\frac{2\ln n}{n^{2}} \right)} = 8 \lim_{n \to \infty} \frac{\ln n}{n^{2}} = 8 \lim_{n \to \infty} \frac{\left(\frac{1}{n} \right)}{\left(\frac{2\ln n}{n^{2}} \right)} = 32 \lim_{n \to \infty} \frac{1}{n^{1/4}} = 32 \cdot 0 = 0$$

15. diverges by the Limit Comparison Test (part 3) with $\frac{1}{n}$, the nth term of the divergent harmonic series: $n\lim_{n\to\infty}\frac{\binom{1}{1-\log n}}{\binom{n}{2}}=n\lim_{n\to\infty}\frac{n}{1-\ln n}=\lim_{n\to\infty}\frac{1}{\binom{n}{2}}=\lim_{n\to\infty}n=\infty$

$$\lim_{n\to\infty} \frac{(\frac{1}{1+\ln n})}{(\frac{1}{n})} = \lim_{n\to\infty} \frac{n}{1+\ln n} = \lim_{n\to\infty} \frac{1}{(\frac{1}{n})} = \lim_{n\to\infty} n = \infty$$

16. diverges by the Limit Comparison Test (part 3) with $\frac{1}{n}$, the nth term of the divergent harmonic series:

$$\lim_{n\to\infty}\frac{\left(\frac{1}{(1+\ln n)^2}\right)}{\binom{1}{n}}=\lim_{n\to\infty}\frac{n}{(1+\ln n)^2}=\lim_{n\to\infty}\frac{1}{\frac{2(1+\ln n)}{[2(1+\ln n)]}}=\lim_{n\to\infty}\frac{n}{2(1+\ln n)}=\lim_{n\to\infty}\frac{\frac{1}{n}}{\binom{2}{n}}=\lim_{n\to\infty}\frac{n}{2}=\infty$$



 $\text{2. converges by the Ratio Test: } \lim_{n \xrightarrow{\longrightarrow} \infty} \frac{a_{n+1}}{a_n} = \lim_{n \xrightarrow{\longrightarrow} \infty} \frac{\left(\frac{(n+1)^2}{e^{n+1}}\right)}{\left(\frac{n^2}{e^n}\right)} = \lim_{n \xrightarrow{\longrightarrow} \infty} \frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2} = \lim_{n \xrightarrow{\longrightarrow} \infty} \left(1 + \frac{1}{n}\right)^2 \left(\frac{1}{e}\right) = \frac{1}{e} < 1$

4. diverges by the Ratio Test: $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{\left(\frac{(n+1)!}{10^{n-1}}\right)}{\left(\frac{n!}{10^{n}}\right)} = \lim_{n \to \infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n \to \infty} \frac{n}{10} = \infty$

8. converges; a geometric series with
$$|\mathbf{r}| = \left| -\frac{2}{3} \right| < 1$$

$$16. \ \ \text{converges by the Ratio Test:} \ \ \underset{n \to \infty}{\text{lim}} \ \ \frac{a_{n+1}}{a_n} = \underset{n \to \infty}{\text{lim}} \ \ \frac{(n+1)\ln{(n+1)}}{2^{n+1}} \cdot \frac{2^n}{n\ln{(n)}} = \frac{1}{2} < 1$$

18. converges by the Ratio Test:
$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^3}{e^{n+1}} \cdot \frac{e^n}{n^3} = \frac{1}{e} < 1$$

In Exercises 45–48, estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series

45.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$
46.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n}$$
47.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.01)^n}{n}$$

46.
$$|error| < |(-1)^6 \left(\frac{1}{10^5}\right)| = 0.00001$$