In Exercises 1–6, find a formula for the nth partial sum of each series and use it to find the series' sum if the series converges.

1. $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^{n-1}} + \cdots$

2. $\frac{9}{100} + \frac{9}{100^2} + \frac{9}{100^3} + \cdots + \frac{9}{100^n} + \cdots$

5. $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{(n+1)(n+2)} + \cdots$

6. $\frac{5}{1 \cdot 2} + \frac{5}{2 \cdot 3} + \frac{5}{3 \cdot 4} + \cdots + \frac{5}{n(n+1)} + \cdots$

In Exercises 7–14, write out the first few terms of each series to show how the series starts. Then find the sum of the series.

2. $s_n = \frac{a(1-r^n)}{(1-r)} = \frac{\left(\frac{9}{100}\right)\left(1-\left(\frac{1}{100}\right)^n\right)}{1-\left(\frac{1}{100}\right)} \Rightarrow \lim_{n \to \infty} s_n = \frac{\left(\frac{9}{100}\right)}{1-\left(\frac{1}{100}\right)} = \frac{1}{11}$

6. $\frac{5}{n(n+1)} = \frac{5}{n} - \frac{5}{n+1} \Rightarrow s_n = \left(5 - \frac{5}{2}\right) + \left(\frac{5}{2} - \frac{5}{3}\right) + \left(\frac{5}{3} - \frac{5}{4}\right) + \ldots + \left(\frac{5}{n-1} - \frac{5}{n}\right) + \left(\frac{5}{n} - \frac{5}{n+1}\right) = 5 - \frac{5}{n+1}$
$\Rightarrow \lim_{n \to \infty} s_n = 5$

Which of the series in Exercises 1–30 converge, and which diverge? Give reasons for your answers. (When you check an answer, remember that there may be more than one way to determine the series' convergence or divergence.)

1. $\sum_{n=1}^{\infty} \frac{1}{10^n}$ **2.** $\sum_{n=1}^{\infty} e^{-n}$ **3.** $\sum_{n=1}^{\infty} \frac{n}{n+1}$

4. $\sum_{n=1}^{\infty} \frac{5}{n+1}$ **5.** $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$ **6.** $\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$

10. $\sum_{n=2} \frac{\ldots}{\sqrt{n}}$ **11.** $\sum_{n=1} \frac{\ldots}{3^n}$ **12.** $\sum_{n=1} \frac{\ldots}{4^n + 3}$

13. $\sum_{n=0}^{\infty} \frac{-2}{n+1}$ **14.** $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ **15.** $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$

16. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ **17.** $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$ **18.** $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

19. $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$ **20.** $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$

16. diverges by the Integral Test: $\int_1^n \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$; $\left[\begin{array}{l} u = \sqrt{x} + 1 \\ du = \frac{dx}{\sqrt{x}} \end{array}\right] \to \int_2^{\sqrt{n}+1} \frac{du}{u} = \ln\left(\sqrt{n}+1\right) - \ln 2$
$\to \infty$ as $n \to \infty$

20. converges; a geometric series with $r = \frac{1}{\ln 3} \approx 0.91 < 1$