Irreducible Quadratic Factors

In Exercises 21–28, express the integrands as a sum of partial fractions and evaluate the integrals.

21. $\displaystyle\int_0^1 \frac{dx}{(x+1)(x^2+1)}$ **22.** $\displaystyle\int_1^{\sqrt{3}} \frac{3t^2+t+4}{t^3+t}\,dt$

$\displaystyle\int \frac{y^2+2y+1}{\ldots}$ $\displaystyle\int \frac{8x^2+8x+2}{\ldots}$

22. $\frac{3t^2+t+4}{t^3+t} = \frac{A}{t} + \frac{Bt+C}{t^2+1} \Rightarrow 3t^2+t+4 = A(t^2+1)+(Bt+C)t;\ t=0 \Rightarrow A=4$; coefficient of t^2

$= A + B \Rightarrow A + B = 3 \Rightarrow B = -1$; coefficient of $t = C \Rightarrow C = 1$; $\int_1^{\sqrt{3}} \frac{3t^2+t+4}{t^3+1}\,dt$

$= 4\int_1^{\sqrt{3}} \frac{dt}{t} + \int_1^{\sqrt{3}} \frac{(-t+1)}{t^2+1}\,dt = \left[4\ln|t| - \frac{1}{2}\ln(t^2+1) + \tan^{-1} t\right]_1^{\sqrt{3}}$

$= \left(4\ln\sqrt{3} - \frac{1}{2}\ln 4 + \tan^{-1}\sqrt{3}\right) - \left(4\ln 1 - \frac{1}{2}\ln 2 + \tan^{-1} 1\right) = 2\ln 3 - \ln 2 + \frac{\pi}{3} + \frac{1}{2}\ln 2 - \frac{\pi}{4}$

$= 2\ln 3 - \frac{1}{2}\ln 2 + \frac{\pi}{12} = \ln\left(\frac{9}{\sqrt{2}}\right) + \frac{\pi}{12}$

ises 33–38.

34. $\displaystyle\int_0^{\pi/2} \sin 2x \cos 3x\,dx$

34. $\int_0^{\pi/2} \sin 2x \cos 3x\,dx = \frac{1}{2}\int_0^{\pi/2} (\sin(-x) + \sin 5x)\,dx = \frac{1}{2}\left[\cos(-x) - \frac{1}{5}\cos 5x\right]_0^{\pi/2} = \frac{1}{2}(0) - \frac{1}{2}\left(1 - \frac{1}{5}\right) = -\frac{2}{5}$

$\int_0 \ (4 - x^2)^{3/2}$

> 1 **22.** $\displaystyle\int \frac{x^2\,dx}{(x^2-1)^{5/2}},\quad x > 1$

22. $x = \sec\theta,\ 0 < \theta < \frac{\pi}{2},\ dx = \sec\theta\tan\theta\,d\theta,\ (x^2-1)^{5/2} = \tan^5\theta;$

$\int \frac{x^2\,dx}{(x^2-1)^{5/2}} = \int \frac{\sec^2\theta \cdot \sec\theta\tan\theta\,d\theta}{\tan^5\theta} = \int \frac{\cos\theta}{\sin^4\theta}\,d\theta = -\frac{1}{3\sin^3\theta} + C = -\frac{x^3}{3(x^2-1)^{3/2}} + C$