

10. (a) There are $(60)(60)(24)(365) = 31,536,000$ seconds in a year. Thus, assuming exponential growth,

$$P = 257,313,431e^{kt} \text{ and } 257,313,432 = 257,313,431e^{(14k/31,536,000)} \Rightarrow \ln\left(\frac{257,313,432}{257,313,431}\right) = \frac{14k}{31,536,000}$$

$$\Rightarrow k \approx 0.0087542$$

(b) $P = 257,313,431e^{(0.0087542)(15)} \approx 293,420,847$ (to the nearest integer). Answers will vary considerably with the number of decimal places retained.

26. It could take 450,000 steps for a sequential search, but at most 19 steps for a binary search because

$$2^{18} = 262,144 < 450,000 < 524,288 = 2^{19}.$$

14. $\alpha = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \sin \alpha = \frac{4}{5}, \cos \alpha = \frac{3}{5}, \sec \alpha = \frac{5}{3}, \csc \alpha = \frac{5}{4}, \text{ and } \cot \alpha = \frac{3}{4}$

$$42. \lim_{x \rightarrow -1^+} \cos^{-1} x = \pi$$

$$6. \sinh(2 \ln x) = \frac{e^{2 \ln x} - e^{-2 \ln x}}{2} = \frac{e^{\ln x^2} - e^{\ln x^{-2}}}{2} = \frac{\left(x^2 - \frac{1}{x^2}\right)}{2} = \frac{x^4 - 1}{2x^2}$$

$$7. \cosh 5x + \sinh 5x = \frac{e^{5x} + e^{-5x}}{2} + \frac{e^{5x} - e^{-5x}}{2} = e^{5x}$$

$$8. \cosh 3x - \sinh 3x = \frac{e^{3x} + e^{-3x}}{2} - \frac{e^{3x} - e^{-3x}}{2} = e^{-3x}$$

$$77. (a) v = \sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{gk}{m}} t\right) \Rightarrow \frac{dv}{dt} = \sqrt{\frac{mg}{k}} \left[\operatorname{sech}^2\left(\sqrt{\frac{gk}{m}} t\right)\right] \left(\sqrt{\frac{gk}{m}}\right) = g \operatorname{sech}^2\left(\sqrt{\frac{gk}{m}} t\right).$$

Thus $m \frac{dv}{dt} = mg \operatorname{sech}^2\left(\sqrt{\frac{gk}{m}} t\right) = mg\left(1 - \tanh^2\left(\sqrt{\frac{gk}{m}} t\right)\right) = mg - kv^2$. Also, since $\tanh x = 0$ when $x = 0$, $v = 0$ when $t = 0$.

$$(b) \lim_{t \rightarrow \infty} v = \lim_{t \rightarrow \infty} \sqrt{\frac{mg}{k}} \tanh\left(\sqrt{\frac{gk}{m}} t\right) = \sqrt{\frac{mg}{k}} \lim_{t \rightarrow \infty} \tanh\left(\sqrt{\frac{gk}{m}} t\right) = \sqrt{\frac{mg}{k}} (1) = \sqrt{\frac{mg}{k}}$$

$$(c) \sqrt{\frac{160}{0.005}} = \sqrt{\frac{160,000}{5}} = \frac{400}{\sqrt{5}} = 80\sqrt{5} \approx 178.89 \text{ ft/sec}$$