40. $y = x^{(x+1)} \Rightarrow \ln y = \ln x^{(x+1)} = (x+1)\ln x \Rightarrow \frac{y'}{y} = \ln x + (x+1)\left(\frac{1}{x}\right) = \ln x + 1 + \frac{1}{x}$

$\Rightarrow y' = x^{(x+1)}\left(1 + \frac{1}{x} + \ln x\right)$

**6. a.** $\overline{\log_3 x}$ **b.** $\overline{\log_{\sqrt{2}} x}$ **c.** $\overline{\log_b a}$

Solve the equations in Exercises 7–10 for $x$.

**7.** $3^{\log_3(7)} + 2^{\log_2(5)} = 5^{\log_5(x)}$

**8.** $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)}$

**9.** $3^{\log_3(x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10}(2)}$

**10.** $\ln e + 4^{-2\log_4(x)} = \frac{1}{2}\log_{10}(100)$

## Logarithmic Differentiation

In Exercises 39–46, use logarithmic differentiation to find the derivative of $y$ with respect to the given independent variable.

**39.** $y = (x+1)^x$

**40.** $y = x^{(x+1)}$

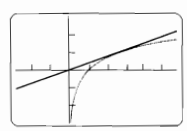
**41.** $y = (\sqrt{t})^t$

**42.** $y = t^{\sqrt{t}}$

90

**7** **Which is bigger, $\pi^e$ or $e^\pi$?** Calculators have taken some of the mystery out of this once-challenging question. (Go ahead and check; you will see that it is a surprisingly close call.) You can answer the question without a calculator, though.

**a.** Find an equation for the line through the origin tangent to the graph of $y = \ln x$.



[−3, 6] by [−3, 3]

**b.** Give an argument based on the graphs of $y = \ln x$ and the tangent line to explain why $\ln x < x/e$ for all positive $x \neq e$.

**c.** Show that $\ln(x^e) < x$ for all positive $x \neq e$.

**d.** Conclude that $x^e < e^x$ for all positive $x \neq e$.

**e.** So which is bigger, $\pi^e$ or $e^\pi$?

(a) The point of tangency is $(p, \ln p)$ and $m_{tangent} = \frac{1}{p}$ since $\frac{dy}{dx} = \frac{1}{x}$. The tangent line passes through $(0, 0) \Rightarrow$ the equation of the tangent line is $y = \frac{1}{p}x$. The tangent line also passes through$(p, \ln p) \Rightarrow \ln p = \frac{1}{p}p = 1 \Rightarrow p = e$, and the tangent line equation is $y = \frac{1}{e}x$.

(b) $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ for $x \neq 0 \Rightarrow y = \ln x$ is concave downward over its domain. Therefore, $y = \ln x$ lies below the graph of $y = \frac{1}{e}x$ for all $x > 0$, $x \neq e$, and $\ln x < \frac{x}{e}$ for $x > 0$, $x \neq e$.

(c) Multiplying by e, $e \ln x < x$ or $\ln x^e < x$.

(d) Exponentiating both sides of $\ln x^e < x$, we have $e^{\ln x^e} < e^x$, or $x^e < e^x$ for all positive $x \neq e$.

(e) Let $x = \pi$ to see that $\pi^e < e^\pi$. Therefore, $e^\pi$ is bigger.

# Webwork 9.1/Problem 3

Solve the differential equation

$$\frac{du}{dt} = e^{2u+5t}.$$

for the initial condition $u(0) = 7$. (In your case the numbers may have differed.)

We separate the variables: $e^{-2u}du = e^{5t}dt$ and integrate (already bringing both constants to one side):

$$-\frac{1}{2}e^{-2u} = \frac{1}{5}e^{5t} + C.$$

Solving for $u$ yields

$$u = -\frac{1}{2}\ln\left(-2 \cdot \left(\frac{1}{5}e^{5t} + C\right)\right)$$

Do not worry about the $-2$ in the argument for the logarithm – we will have $C$ a big negative number such that the argument of the logarithm is positive!

Now we use the initial condition to get the value for $C$. Setting $t = 0$ yields:

$$7 = u(0) = -\frac{1}{2}\ln\left(-2 \cdot \left(\frac{1}{5}e^{5\cdot 0} + C\right)\right)$$

and therefore

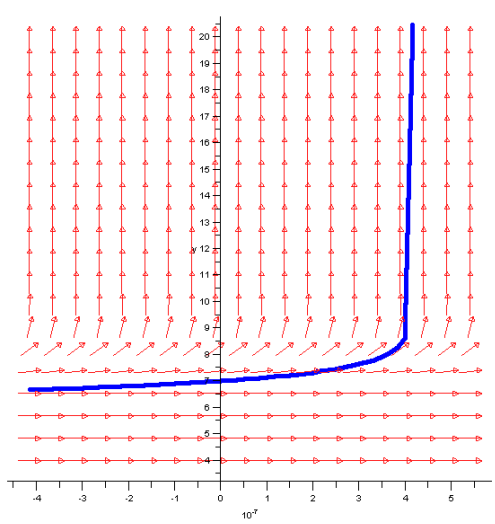
$$C = -\frac{1}{2}e^{-2\cdot 7} - \frac{1}{5}e^{5\cdot 0} = -\frac{1}{2}e^{-14} - \frac{1}{5}.$$

Thus the solution we want is:

$$u = -\frac{1}{2}\ln\left(-2 \cdot \left(\frac{1}{5}e^{5t} - \frac{1}{2}e^{-14} - \frac{1}{5}\right)\right).$$

We see from this solution that for $t > \frac{1}{5}\ln\left(\frac{5}{2e^{14}} + 1\right) \sim 4.1 \cdot 10^{-7}$ the argument of the logarithm becomes negative. This means that for larger $t$ values the differential equation has no solution.

This is not surprising if we plot the solution — we see that at this $t$-value the solution curve goes to $\infty$.




If the differential equation came from a model in physics, this essentially means that the model does not work for larger $t$-values for the given initial configuration $u(0) = 7$. (For example at this point the machine will have been broken or the particle has moved away or hit a wall.)