Replace the polar equations in Exercises 23–48 by equivalent Cartesian equations. Then describe or identify the graph.

23. $r \cos \theta = 2$

24. $r \sin \theta = -1$

25. $r \sin \theta = 0$

26. $r \cos \theta = 0$

27. $r = 4 \csc \theta$

28. $r = -3 \sec \theta$

29. $r \cos \theta + r \sin \theta = 1$

30. $r \sin \theta = r \cos \theta$

31. $r^2 = 1$

32. $r^2 = 4r \sin \theta$

33. $r = \dfrac{5}{\sin \theta - 2 \cos \theta}$

34. $r^2 \sin 2\theta = 2$

35. $r = \cot \theta \csc \theta$

36. $r = 4 \tan \theta \sec \theta$

37. $r = \csc \theta \, e^{r \cos \theta}$

38. $r \sin \theta = \ln r + \ln \cos \theta$

39. $r^2 + 2r^2 \cos \theta \sin \theta = 1$

40. $\cos^2 \theta = \sin^2 \theta$

41. $r^2 = -4r \cos \theta$

42. $r^2 = -6r \sin \theta$

43. $r = 8 \sin \theta$

44. $r = 3 \cos \theta$

45. $r = 2 \cos \theta + 2 \sin \theta$

46. $r = 2 \cos \theta - \sin \theta$

47. $r \sin \left(\theta + \frac{\pi}{6}\right) = 2$

48. $r \sin \left(\frac{2\pi}{3} - \theta\right) = 5$

Cartesian to Polar Equations

Replace the Cartesian equations in Exercises 49–62 by equivalent polar equations.

49. $x = 7$

50. $y = 1$

51. $x = y$

52. $x - y = 3$

53. $x^2 + y^2 = 4$

54. $x^2 - y^2 = 1$

55. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

56. $xy = 2$

58. $x^2 + xy + y^2 = 1$

60. $(x - 5)^2 + y^2 = 25$

26. $r \cos \theta = 0 \Rightarrow x = 0$, the y-axis

$\Rightarrow y = 4$, a horizontal line through $(0, 4)$

30. $r \sin \theta = r \cos \theta \Rightarrow y = x$, line with slope $m = 1$ and intercept $b = 0$

56. $xy = 2 \Rightarrow (r \cos \theta)(r \sin \theta) = 2 \Rightarrow r^2 \cos \theta \sin \theta = 2 \Rightarrow 2r^2 \cos \theta \sin \theta = 4 \Rightarrow r^2 \sin 2\theta = 4$

58. $x^2 + xy + y^2 = 1 \Rightarrow x^2 + y^2 + xy = 1 \Rightarrow r^2 + r^2 \sin \theta \cos \theta = 1 \Rightarrow r^2 (1 + \sin \theta \cos \theta) = 1$

Identify the symmetries of the curves in Exercises 1–12. Then sketch the curves.

1. $r = 1 + \cos\theta$ **2.** $r = 2 - 2\cos\theta$
3. $r = 1 - \sin\theta$ **4.** $r = 1 + \sin\theta$
5. $r = 2 + \sin\theta$ **6.** $r = 1 + 2\sin\theta$
7. $r = \sin(\theta/2)$ **8.** $r = \cos(\theta/2)$
9. $r^2 = \cos\theta$ **10.** $r^2 = \sin\theta$
11. $r^2 = -\sin\theta$ **12.** $r^2 = -\cos\theta$

Graph the lemniscates in Exercises 13–16. What symmetries do these curves have?

13. $r^2 = 4\cos 2\theta$ **14.** $r^2 = 4\sin 2\theta$
15. $r^2 = -\sin 2\theta$ **16.** $r^2 = -\cos 2\theta$

Slopes of Polar Curves

Find the slopes of the curves in Exercises 17–20 at the given points. Sketch the curves along with their tangents at these points.

17. Cardioid $r = -1 + \cos\theta;\quad \theta = \pm\pi/2$
18. Cardioid $r = -1 + \sin\theta;\quad \theta = 0, \pi$
19. Four-leaved rose $r = \sin 2\theta;\quad \theta = \pm\pi/4, \pm 3\pi/4$
20. Four-leaved rose $r = \cos 2\theta;\quad \theta = 0, \pm\pi/2, \pi$

Limaçons

Graph the limaçons in Exercises 21–24. Limaçon ("*lee*-ma-sahn") is Old French for "snail." You will understand the name when you graph

a. $r = \frac{3}{2} + \cos\theta$ **b.** $r = \frac{3}{2} - \sin\theta$

24. Oval limaçons

a. $r = 2 + \cos\theta$ **b.** $r = -2 + \sin\theta$

Graphing Polar Inequalities

25. Sketch the region defined by the inequalities $-1 \le r \le 2$ and $-\pi/2 \le \theta \le \pi/2$.

26. Sketch the region defined by the inequalities $0 \le r \le 2\sec\theta$ and $-\pi/4 \le \theta \le \pi/4$.

In Exercises 27 and 28, sketch the region defined by the inequality.

27. $0 \le r \le 2 - 2\cos\theta$ **28.** $0 \le r^2 \le \cos\theta$

Intersections

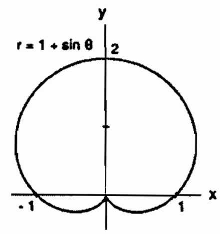
29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2\sin 2\theta$.

30. Show that $(1/2, 3\pi/2)$ lies on the curve $r = -\sin(\theta/3)$.

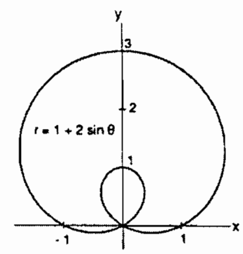
Find the points of intersection of the pairs of curves in Exercises 31–38.

31. $r = 1 + \cos\theta,\quad r = 1 - \cos\theta$
32. $r = 1 + \sin\theta,\quad r = 1 - \sin\theta$
33. $r = 2\sin\theta,\quad r = 2\sin 2\theta$
34. $r = \cos\theta,\quad r = 1 - \cos\theta$
35. $r = \sqrt{2},\quad r^2 = 4\sin\theta$
36. $r^2 = \sqrt{2}\sin\theta,\quad r^2 = \sqrt{2}\cos\theta$
37. $r = 1,\quad r^2 = 2\sin 2\theta$

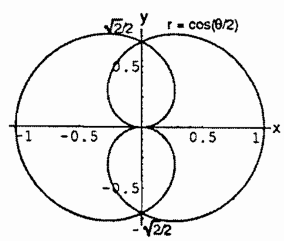
4. $1 + \sin(-\theta) = 1 - \sin\theta \ne r$ and $1 + \sin(\pi - \theta)$
$= 1 + \sin\theta \ne -r \Rightarrow$ not symmetric about the x-axis;
$1 + \sin(\pi - \theta) = 1 + \sin\theta = r \Rightarrow$ symmetric about the y-axis; therefore not symmetric about the origin



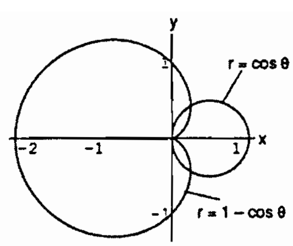
6. $1 + 2\sin(-\theta) = 1 - 2\sin\theta \ne r$ and $1 + 2\sin(\pi - \theta)$
$= 1 + 2\sin\theta \ne -r \Rightarrow$ not symmetric about the x-axis;
$1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta = r \Rightarrow$ symmetric about the y-axis; therefore not symmetric about the origin



8. $\cos\left(-\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = r \Rightarrow$ symmetric about the x-axis;
$\cos\left(\frac{2\pi - \theta}{2}\right) = \cos\left(\frac{\theta}{2}\right)$, so the graph is symmetric about the y-axis, and hence the origin.



34. $\cos\theta = 1 - \cos\theta \Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{3}, -\frac{\pi}{3} \Rightarrow r = \frac{1}{2}$; points of intersection are $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ and $\left(\frac{1}{2}, -\frac{\pi}{3}\right)$. The point $(0, 0)$ is found by graphing.



36. $\sqrt{2}\sin\theta = \sqrt{2}\cos\theta \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$;
$\theta = \frac{\pi}{4} \Rightarrow r^2 = 1 \Rightarrow r = \pm 1$ and $\theta = \frac{5\pi}{4} \Rightarrow r^2 = -1$
$\Rightarrow$ no solution for r; points of intersection are $\left(\pm 1, \frac{\pi}{4}\right)$.
The points $(0, 0)$ and $\left(\pm 1, \frac{3\pi}{4}\right)$ are found by graphing.