1. Match the Taylor series with the functions. **Place the number (Roman numercal) of the appropriate series in the center column, to the right of the given function.**

A. $\cos(2x^2)$	(ii)	(i) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n x^{2n}}{n!}$
B. $\ln(1+2x^2)$	(iii)	(ii) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n)!}x^{4n}$
C. e^{-2x^2}	(i)	(iii) $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n}x^{4n}$
D. $\sin(2x^2)$	(iv)	(iv) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n+1}x^{4n+2}}{(2n+1)!}$
E. $\frac{1}{1+2x^2}$	(v)	(v) $\sum_{n=0}^{\infty}(-1)^n 2^n x^{2n}$

3 each)
no pc

A.______ B.______ C.______ D.______ E.______

2. Determine whether the series is convergent or divergent. In either case explain why. **You must justify your answers.**

(a) $\sum_{n=1}^{\infty} \frac{10^n}{9^n}$

$\lim_{n\to\infty} \frac{10^n}{9^n} = \infty$, 5 div by Test for div.

3

Div. because it is a geom. series with

$r = \frac{10}{9} > 1$. 2

$\frac{a_{n+1}}{a_n} = \frac{10^{n+1}/9^{n+1}}{10^n/9^n} = \frac{10}{9} > 1$ 3 div 2

Same with root

(b) $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$

$b_n = \frac{1}{n^{3/2}}$ 1, Σb_n conv, p series $p = \frac{3}{2}$ 1

$\frac{b_n}{\frac{n+1}{n^2\sqrt{n}}} = \frac{\frac{1}{n^{3/2}}}{\frac{n+1}{n^2\sqrt{n}}} = \frac{n^{5/2}}{n^{5/2}+n^{3/2}} = \frac{1}{1+\frac{1}{n}} \to 1$ as $n \to \infty$ 2 1

$\therefore \sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ converges by LCT

-1 if not there.

(c) $\sum_{n=1}^{\infty} \frac{9^n}{10^n} = \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$

3

Converges. Geom. series $r = \frac{9}{10} < 1$ 2

$\frac{a_{n+1}}{a_n} = \frac{(9/10)^{n+1}}{(9/10)^n} = \frac{9}{10} \to \frac{9}{10} < 1$ 3 as $n \to \infty$.

Converges by ratio test 2

Same for root test.

3. Determine whether the series is absolutely convergent, conditionally convergent or divergent. In any case, **explain why.**

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{\ln(n^2+2n+6)}$

$$\lim_{x\to\infty} \frac{\ln x}{\ln(x^2+2x+6)} \overset{LH}{=} \lim_{x\to\infty} \frac{1/x}{\frac{2x+2}{x^2+2x+6}}$$

$$= \lim_{x\to\infty} \frac{x^2+2x+6}{x(2x+2)} = \lim_{x\to\infty} \frac{1+\frac{2}{x}+\frac{6}{x^2}}{2+\frac{2}{x}} = \frac{1}{2} \quad 3$$

$\therefore$ diverges by test for div. 2

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$

1. alt: $(-1)^n$

2, $a_n = \dfrac{\pi^{2n}}{(2n)!}$ $\qquad a_{n+1} = \dfrac{\pi^{2(n+1)}}{(2n+2)!}$

$$= \frac{\pi^{2n}\pi^2}{(2n+2)(2n+1)(2n)!}$$

$$= a_n \frac{\pi^2}{(2n+2)(2n+1)}$$

$< a_n$ for $n \geq 1$.

Some p.c. if they do some of this stuff correctly

3. $\displaystyle\lim_{n\to\infty} \frac{\pi^{2n}}{(2n)!} = 0$ by form 5, front page.

$\therefore$ converges by AST,

all that is nec.

Consider $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{2n}}{(2n)!}$, $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\pi^{2n+2}/(2n+2)!}{\pi^{2n}/(2n)!}\right| = \frac{\pi^2}{(2n+2)(2n+1)} \to 0$ 2

Converges absolutely 1 2

(c) If the series in part (b) converges, find the sum.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!} = \cos\pi = -1.$$

5

4. For what values of x does the following power series converge: $\sum_{n=0}^{\infty}(-1)^n \frac{(3x)^n}{n^2+3}$. (You must show all of your work.)

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(3x)^{n+1}/(n+1)^2+3}{(3x)^n/(n^2+1)}\right| = \frac{n^2+1}{(n+1)^2+3}|3x| = \frac{1+\frac{1}{n^2}}{(1+\frac{1}{n})^2+\frac{3}{n^2}}|3x|$$

$$\to |3x| < 1$$

Converges for $|x| < \frac{1}{3}$ or $(-\frac{1}{3}, \frac{1}{3})$. 3

$x = \frac{1}{3}$: $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2+3}$ 1 . $\sum_{n=0}^{\infty} \frac{1}{n^2+3}$ converges 1

So $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2+3}$ converges by abs. conv.

1 $\sum_{n=0}^{\infty} \frac{1}{n^2+3}$ converges because $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges p series $p=2$

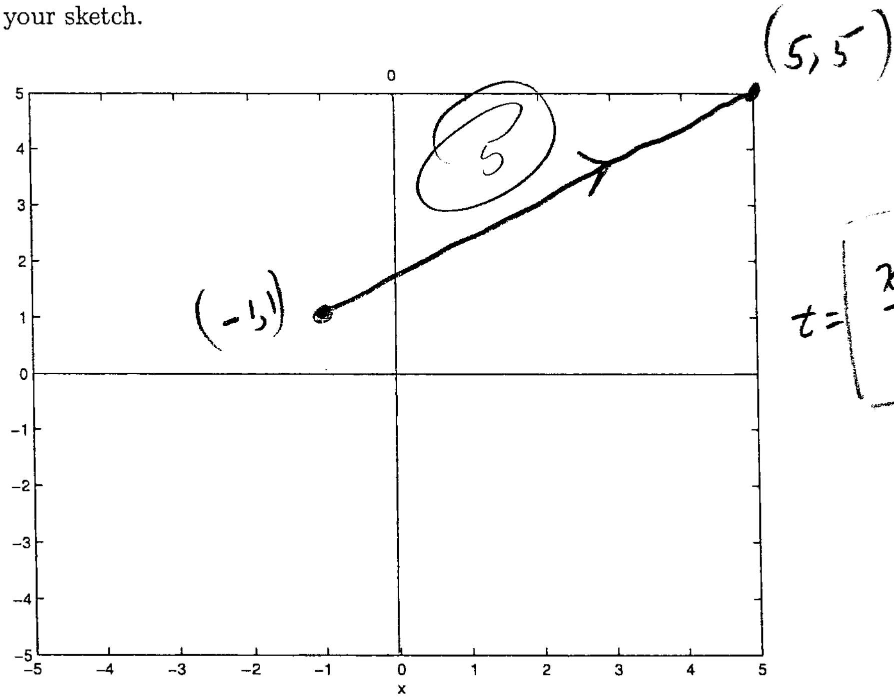
$$\frac{\frac{1}{n^2}}{\frac{1}{n^2+3}} = \frac{n^2+3}{n^2} = \frac{1+3/n^2}{1} \to 1 .$$ So $\sum_{n=0}^{\infty} \frac{1}{3+n^2}$

converges by LCT.

$x = -\frac{1}{3}$: 1 $\sum_{n=0}^{\infty} (-1)^n \frac{(-1)^n}{n^2+3} = \sum_{n=0}^{\infty} \frac{1}{n^2+3}$ 1 converges by 1

$\therefore$ Converges on $[-1/3, 1/3]$ or $-\frac{1}{3} \leq x \leq \frac{1}{3}$

5. (a) Sketch the curve represented by the parametric equation $x = 3t - 1$, $y = 2t + 1$, $0 \le t \le 2$ and write the corresponding rectangular/Cartesian equation for the curve. Make sure to include the direction of motion in your sketch.



$t = \boxed{\dfrac{x+1}{3} = \dfrac{y-1}{2}}$

5

(b) Find the length of the parametric curve $x = \sqrt{t}$, $y = 3t - 1$, $0 \le t \le 1$. You do not have to evaluate the integral.

$x' = \dfrac{1}{2\sqrt{x}}$, $y' = 3$, $L = \displaystyle\int_0^1 \sqrt{\dfrac{1}{4x} + 9}\, dt$.

5

6. Show that the Maclaurin series for the function $f(x) = \cos(7x)$,

$$T(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(7x)^{2n}}{(2n)!} = 1 - \frac{7^2x^2}{2!} + \frac{7^4x^4}{4!} - \frac{7^6x^6}{6!} + \cdots,$$

(which converges for all real x) converges to $f(x)$, i.e. show that $T(x) = f(x)$.

$$\cos(7x) = T_n(x) + R_n(x) = \sum_{k=0}^{n} (-1)^k \frac{(7x)^{2k}}{(2k)!} + \frac{(-1)^n}{n!} \int_0^x (t-x)^n f^{(n+1)}(t)\, dt$$

$$f^{(n+1)}(t) = \cancel{\pm 7^{n+1} \sin t} \text{ or } \pm 7^{n+1} \cos t.$$

2

In any of the cases $|f^{(n+1)}(t)| \leq 7^{n+1}$

$M = 7^{n+1}$

$$\therefore\ |R_n(x)| \leq \frac{1}{n!} \left| \int_0^x |(t-x)^n| \, |f^{(n+1)}(t)|\, dt \right|$$

2

$$\leq \frac{1}{n!} \left| \int_0^x |(t-x)^n| \cdot 1 \, dt \right|$$

$$= \frac{1}{n!} \left[\frac{|(t-x)^{n+1}|}{n+1} \right]_0^x = \frac{7^{n+1}}{(n+1)!} |x|^{n+1} \to 0 \text{ as } n \to \infty \text{ for any}$$

2 (by form 5, from

By sandwich Thm (or definition) $R_n(x) \to 0$ as $n \to \infty$ for any x.

$$\therefore\ \cos(7x) = \lim_{n\to\infty} \left(T_n(x) + R_n(x)\right) = \lim_{n\to\infty} T_n(x) + 0$$

$$= \lim_{n\to\infty} T_n(x)$$

$$= T(x).$$

7. Use power series to solve the differential equation $y' = \pi y$ with initial value $y(0) = 1$.

$$y = \sum_{n=0}^{\infty} a_n x^n, \qquad y' = \sum_{n=1}^{\infty} a_n n x^{n-1}.$$

2

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$$

$$= \pi y = \pi a_0 + \pi a_1 x + \pi a_2 x^2 + \pi a_3 x^3 + \cdots$$

$$\Rightarrow \quad a_1 = \pi a_0 = \frac{1}{1!}\pi^1 a_0$$

$$2a_2 = \pi a_1 \text{ or } a_2 = \frac{1}{2}\pi a_1 = \frac{1}{2}\pi^2 a_0 = \frac{1}{2!}\pi^2 a_0$$

2

$$3a_3 = \pi a_2 \text{ or } a_3 = \frac{1}{3}\pi a_2 = \frac{1}{3\cdot 2}\pi^3 a_0 = \frac{1}{3!}\pi^3 a_0$$

$$4a_4 = \pi a_3 \text{ or } a_4 = \frac{1}{4}\pi a_3 = \frac{1}{4!}\pi^4 a_0$$

$$a_n = \frac{1}{n!}\pi^n a_0 \qquad n \geq 1.$$

2

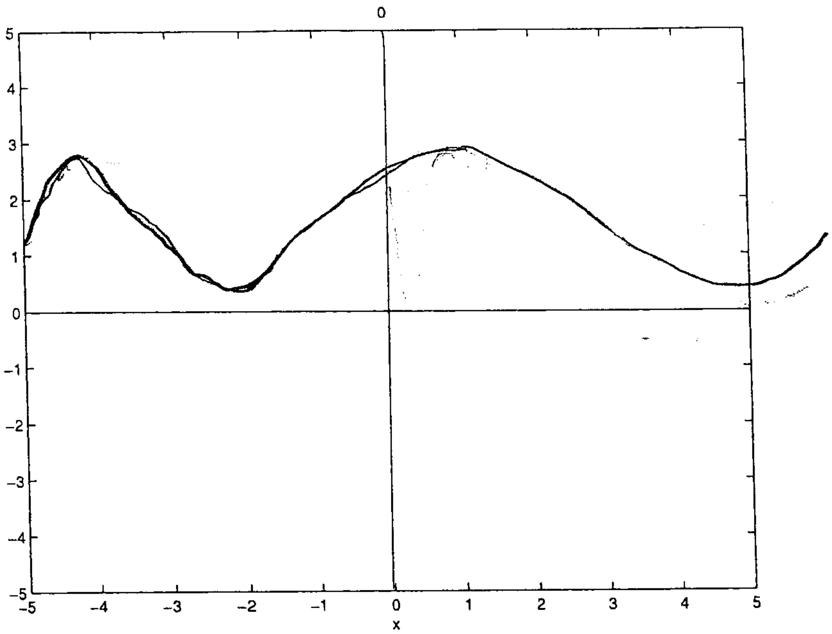
$$\therefore \quad y(x) = a_0 \sum_{n=0}^{\infty} \frac{1}{n!}\pi^n x^n$$

$$y(0) = 1 = a_0 \cdot 1 \qquad \text{So } a_0 = 1.$$

2

$$\therefore \quad y(x) = \sum_{n=0}^{\infty} \frac{1}{n!}\pi^n x^n.$$

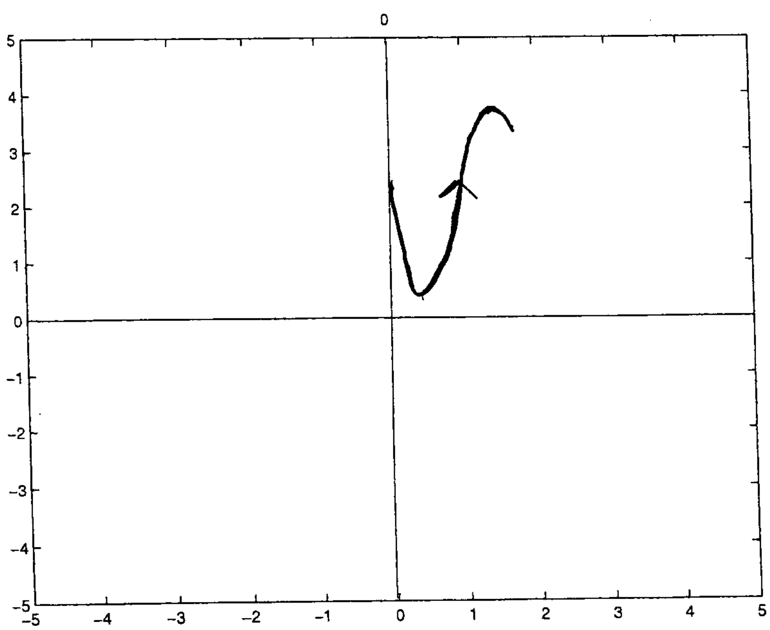
8. On the axes given below plot the function $y = e^{\sin x}$.



5

−1 for bad thing, ie. way too high, too low, periodic, etc

9. Plot the parametric curve $x = \frac{1}{2}e^{t/2}$, $y = e^{\sin t}$, $-4 \le t \le 2$. Indicate the direction by arrows that the curve is traversed as t goes from ~~0 to 3π~~ −4 to 2.



5

−2 for wrong or no direction

−1 for bad thing, ie. bad starting pt, bad ending pt, not low enough, etc

$x = \frac{2}{3} t^{3/2}, \quad y = \frac{1}{6}(3t-1)^2, \quad 1 \le t \le 3$

10. Suppose for the parametric curve $x =$ ~~[illegible]~~ we found the following expression for the length of the curve: $L = \int_{1}^{3} \sqrt{t + (3t-1)^2}\, dt$. Find L.

~~14.11308861350159~~

$= 10.4005268225884$ +5

5