

8) A scientist obtained experimental data that leads to the following system of equations.

$$\begin{array}{rclcl} 1x+ & 2y+ & 3z & = & 5 \\ 4x+ & 5y+ & 6z & = & 7 \\ 7x+ & 8y+ & 10z & = & 14 \\ 9x+ & 15y+ & 21z & = & 31 \end{array}$$

Since more measurements than necessary were made, this system is overdetermined. Because of small measurement errors it has no solutions.

Determine the best approximation of a solution. (This is the solution to the equation $Ax = b'$ where b' is the orthogonal projection of b on the column space of A .)

9) a) How many operations (count multiplication, addition, subtraction, comparisons each as a cost 1 operation) are required to invert a $n \times n$ matrix (assume that entries that were not cleared out are always nonzero so that no shortcut is possible)?

b) How many operations are needed to do an LU factorization of an $n \times n$ matrix?

c) How many operations are needed to do an PLU factorization (including partial pivoting)?

d) How many operations are required with full pivoting?

10) This problem should be solved using single precision floating point numbers.

a) Write a C routine that will multiply a vector and a matrix, given in the format used by LAPack

b) Write a program that uses LAPack routines to compute the inverse of a matrix (for example by solving for all columns of the identity matrix).

c) The $n \times n$ Hilbert matrix is the matrix $(a_{i,j})$ with $a_{i,j} = \frac{1}{i+j-1}$. Let A be the 10×10 Hilbert matrix and $\underline{\mathbf{b}} = (1, 0, \dots, 0)^T$. Using LAPack, solve the equation $A\underline{\mathbf{x}} = \underline{\mathbf{b}}$.

Compute (using the routine in a) for matrix multiplication) the difference $A\underline{\mathbf{x}} - \underline{\mathbf{b}}$.

d) Using the inversion routine from b), compute for the same A and $\underline{\mathbf{b}}$ the solution $\underline{\mathbf{x}}_2 := A^{-1}\underline{\mathbf{b}}$.

Again, compute the difference $A\underline{\mathbf{x}}_2 - \underline{\mathbf{b}}$.

e) Compare the differences obtained in c) and d). What can we deduce about the numerical stability of inverting a matrix.