Number Systems

\( \mathbb{N} \): Set of Natural Numbers, 1, 2, 3,\ldots
\( \mathbb{Z} \): Set of Integers, 0, \pm 1, \pm 2, \pm 3,\ldots
\( \mathbb{Q} \): Set of Rational Numbers, \( \frac{m}{n} \), where \( m, n \) are integers (with no common divisor) and \( n \neq 0 \)
\( \mathbb{R} \): Set of Real Numbers

Set Notation (see Text, Appendix)

\( x \in S \): where \( S \) is a set of objects (normally numbers), means: “\( x \) is an element of \( S \)”
\( x \notin S \): \( x \) is “some element” but not in \( S \)

Examples:
\( 1 \in \mathbb{N} \), \( \frac{1}{2} \in \mathbb{Q} \), \( \frac{1}{2} \notin \mathbb{N} \), \( \sqrt{2} \in \mathbb{R} \), \( \sqrt{2} \notin \mathbb{Q} \).
\( S \subset T \): \( S \) is a (true) subset of \( T \)
\( S \subseteq T \): \( S \subset T \) or \( S = T \)

Containments of number sets: \( \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \)

Specifying Sets: (Notation: \{variable : property\})

\( \{1,2,3,4,5\} = \{n : n \in \mathbb{N} \text{ and } n \leq 5\} = \{n \in \mathbb{N} : n \leq 5\} \)

\( \{1,3,5,7,\ldots\} = \{n \in \mathbb{N} : n \text{ is odd}\} = \{n : n = 2m - 1 \text{ for some } m \in \mathbb{N}\} \)

Set Operations: (with two sets \( S \) and \( T \))

\( S \setminus T = \{x \in S : x \notin T\} \)
\( S \cap T = \{x : x \in S \text{ and } x \in T\} \)
\( S \cup T = \{x : x \in S \text{ or } x \in T\} \)

\( \{1,2\} \cap \{3,4\} = \emptyset \), where \( \emptyset \) is the empty set that contains no element.

Note: \( \emptyset \subseteq S \) for any set \( S \).

Notation from Logic

\( A \Rightarrow B \): “\( A \) implies \( B \)” or “If \( A \) holds then \( B \) holds” or “\( B \) is a necessary condition for \( A \)” or “\( A \) is a sufficient condition for \( B \)”

\( A \iff B \): “\( A \) implies \( B \) and \( B \) implies \( A \)” or “\( B \) holds if and only if \( A \) holds” or “\( B \) is a necessary and sufficient condition for \( A \)”

\( \forall \): ”For all” (e.g. \( \forall n \in \mathbb{N} : n > 0 \))

\( \exists \): “There exists” (e.g.: If \( p, q \in \mathbb{Q} \) and \( p < q \), then \( \exists r \in \mathbb{R} \) such that \( p < r < q \)