

We have discussed:

- ✧ Voting Theory
- ✧ Arrow's Impossibility Theorem
- ✧ Voting Methods:
 - ◆ Plurality
 - ◆ Borda Count
 - ◆ Plurality with Elimination
 - ◆ Pairwise Comparisons

- ✧ Voting Criteria:
 - ◆ Majority Criterion
 - ◆ Condorcet Criterion
 - ◆ Monotonicity Criterion
 - ◆ Independence of Irrelevant Alternatives Criterion

Who gets second? Third? etc.

Rankings

The MAS is electing president, vice president and treasurer.

Let first choice be president, second choice the vice president, etc.

Who wins which position under the plurality method?

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

Office	Place	Candidate	# first place votes
President	First	A	14
Vice	Second	C	11
Treasurer	Third	D	8
	Fourth	B	4

Call ranking methods “Extended”

Use Extended Borda Count to assign positions for the MAS election.

Borda Count results:

$$\mathbf{A: 14 \times 4 + 10 \times 1 + 8 \times 1 + 4 \times 1 + 1 \times 1 = 79}$$

$$\mathbf{B: 14 \times 3 + 10 \times 3 + 8 \times 2 + 4 \times 4 + 1 \times 2 = 106}$$

$$\mathbf{C: 14 \times 2 + 10 \times 4 + 8 \times 3 + 4 \times 2 + 1 \times 4 = 104}$$

$$\mathbf{D: 14 \times 1 + 10 \times 2 + 8 \times 4 + 4 \times 3 + 1 \times 3 = 81}$$

Ranking: B,C,D,A

Use Extended Plurality-with-Elimination to assign positions for the MAS election.

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

Ranking: D, A, C, B

Try Extended Pairwise Comparison on MAS

example:

From previous work:

A	B	A	C	A	D
14	10	14	10	14	10
	8		8		8
	4		4		4
	1		1		1

B	C	B	D	C	D
14	10	14	8	14	8
4	8	10	1	10	4
	1		4		1

Ranking: C, B, D, A

Try Extended Plurality-with-Elimination on the

Where to Eat Dinner? example.

Preference Schedule: Where to eat?

Number of voters	4	2	5
First choice	A	B	C
Second choice	B	C	A
Third choice	C	A	B

Ranking: C, A, B

Recursive Ranking Methods

Ranks candidates by using the chosen voting method to find each place.

Step 1: Decide on a voting method

Step 2: Use the method to find the winner- this candidate gets first place

Step 3: Make a new preference schedule without the first place candidate.

Step 4: Use the modified preference schedule and chosen voting method to find the winner- this candidate gets second place.

Step 5: Make a new preference schedule without the first or second place candidates.

Repeat as needed until all candidates have been ranked.

MAS Example

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

Recursive Plurality Method:

✧Step 1: Note- use Plurality

✧Step 2: A wins

✧Step 3: Preference schedule without A

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	B	C	D	B	C
Second choice	C	B	C	D	D
Third choice	D	D	B	C	B

✧Step 4: Find winner by plurality: B

✧Step 5: Remove B from schedule:

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	C	C	D	D	C
Second choice	D	D	C	C	D

✧Step 6: Find winner by plurality.(last time)

C has 25 votes, D has 12.

Ranking:

First Place: A

Second Place: B

Third Place: C

Fourth Place: D

Try Recursive Pairwise:

A	B
14	10
	8
	4
	1

A	C
14	10
	8
	4
	1

A	D
14	10
	8
	4
	1

B	C
14	10
4	8
	1

B	D
14	8
10	1
4	

C	D
14	8
10	4
1	

C is winner, so remove C from the schedule.

Count again, and B wins. Remove B.

Between D and A, D wins.

Ranking for MAS Recursive Pairwise:

First: C

Second: B

Third: D

Fourth: A

Recursive Plurality-with-Elimination

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

Plurality with Elimination: Eliminate the candidates with the fewest first place votes until one candidate has a majority. (19 votes in this example)

From Previous Work, D is winner.

Recursive Plurality with Elimination: Remove D as first place, do the method again to find second place.

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	C	B	C
Second choice	B	B	B	C	B
Third choice	C	A	A	A	A

Do plurality with elimination again.

Notice C has 19 votes, a majority. C wins second place overall.

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	B	B	B	B
Second choice	B	A	A	A	A

B wins this round, taking third overall. A is last.

Ranking:

First: D

Second: C

Third: B

Fourth: A

Try Recursive Plurality-with-Elimination on the Favorite Fruit example.

Preference Schedule: Favorite Fruit

Number of voters	2	2	1
First choice	M	B	M
Second choice	P	M	A
Third choice	A	A	P
Fourth choice	B	P	B

Round 1: M has majority, gets first place.

Round 2: Modified preference schedule

Preference Schedule: Favorite Fruit

Number of voters	2	2	1
First choice	P	B	A
Second choice	A	A	P
Third choice	B	P	B

No one has majority, A has fewest first place votes.

With A removed, P has a majority and takes second place.

Preference Schedule: Favorite Fruit

Number of voters	2	2	1
First choice	A	B	A
Second choice	B	A	B

A has majority, takes third. B is last.

Ranking

First: M

Second: P

Third: A

Fourth: B

Chapter One Summary

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- ✧ Arrow's Impossibility Theorem
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 - ◆ Plurality with Elimination
 - ◆ Pairwise Comparisons
 - ◆ Extended Plurality
 - ◆ Extended Borda Count
 - ◆ Extended Plurality with Elimination
 - ◆ Extended Pairwise Comparisons
 - ◆ Recursive Plurality
 - ◆ Recursive Borda Count
 - ◆ Recursive Pairwise Comparisons
 - ◆ Recursive Plurality with Elimination

- ✧ Voting Criteria:
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