Voting Methods discussed thus far:

Plurality

Borda Count

Note: neither require a majority to select a winner

What if we Need a Majority?

Plurality-with-elimination

✦ **Also called Instant Runoff Voting**

✦ **Guarentees winner has a majority of the votes**

✦ **Eliminates low-vote candidates**

✦ **Preference ballots- no need to run multiple elections**

Round One

Count first place votes. Check for majority; if there is, declare the winner and you are finished. If no majority, eliminate the candidate with the least number of first place votes.

Round Two

Recount the first place votes.

Check for majority and winner.

Eliminate.

Rounds Three and so on

Repeat until a candidate has a majority.

Try plurality-with-elimination on the MAS

Example:

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

Round One

Count first place votes:

A: 14, B: 4, C: 11, D: 8

Eliminate candidate B and rewrite the preference schedule:

Preference Schedule: MAS Election(w/o B)

Number of voters	14	10	8	4	1
First choice	A	C	D	D	C
Second choice	C	D	C	C	D
Third choice	D	A	A	A	A

Round Two

Count first place votes:

A: 14, C: 11, D: 12

Eliminate candidate C and rewrite the preference schedule:

Preference Schedule: MAS Election(w/o B and C)					
Number of voters	14	10	8	4	1
First choice	A	D	D	D	D
Second choice	D	A	A	A	A

Round Three

Count first place votes:

A: 14, D: 23

We are done, and D is the winner.

What can go wrong with Plurality-with-Elimination?

Which city should host the Olympics?

(Athens, Babylon, Carthage)

Preference Schedule: Olympics, Straw Poll				
Number of voters	7	8	10	4
First choice	A	B	C	A
Second choice	B	C	A	C
Third choice	C	A	B	B

No candidate has majority, drop B.

Preference Schedule: Olympics, Straw Poll				
Number of voters	7	8	10	4
First choice	A	C	C	A
Second choice	C	A	A	C

C has majority, is winner.

As a result of the straw poll, 4 people change their first place vote from A to C, giving C more first place votes.

Preference Schedule, real election:

Preference Schedule: Olympics			
Number of voters	7	8	14
First choice	A	B	C
Second choice	B	C	A
Third choice	C	A	B

No majority, drop A.

Preference Schedule: Olympics			
Number of voters	7	8	14
First choice	B	B	C
Second choice	C	C	B

B has majority and wins!

Monotonicity Criterion

> **If choice *X* is the winner of an election and, in a re-election, the only changes in the ballots are changes that favor *X*, then *X* should remain a winner of the election.**

Method of Pairwise Comparisons

Compare each candidate one-on-one. A win is worth one point, a tie worth $\frac{1}{2}$ point, and a loss worth nothing.

Make tables comparing each candidate:

For example:

A	B

Repeat for all combinations.

Math Appreciation Society Example:

Preference Schedule: MAS Election

Number of voters	14	10	8	4	1
First choice	A	C	D	B	C
Second choice	B	B	C	D	D
Third choice	C	D	B	C	B
Fourth choice	D	A	A	A	A

A	B
14	10
	8
	4
	1

A	C
14	10
	8
	4
	1

A	D
14	10
	8
	4
	1

B	C
14	10
4	8
	1

B	D
14	8
10	1
4	

C	D
14	8
10	4
1	

Add up all the points. (1 for win, $\frac{1}{2}$ for tie)

A: 0, B: 2, C: 3, D:1

C is the winner!

Try the Method of Pairwise Comparison on the fruit example:

Preference Schedule: Favorite Fruit

Number of voters	2	2	1
First choice	M	B	M
Second choice	P	M	A
Third choice	A	A	P
Fourth choice	B	P	B

Use the Method of Pairwise Comparison on the following example:
The Los Angeles LAXers are getting number one choice in upcoming draft. The list is narrowed down to Allen, Byers, Castillo, Dixon and Evans.

Preference Schedule: Laxers Draft

Number of voters	2	6	4	1	1	4	4
First choice	A	B	B	C	C	D	E
Second choice	D	A	A	B	D	A	C
Third choice	C	C	D	A	A	E	D
Fourth choice	B	D	E	D	B	C	B
Fifth choice	E	E	C	E	E	B	A

Set up comparision tables:

A	B

A	C

A	D

A	E

B	C

B	D

B	E

C	D

C	E

D	E

Point totals:

A gets 3 points.

B gets $2\frac{1}{2}$ points.

C gets 2 points.

D gets $1\frac{1}{2}$ points.

E gets 1 point.

Allen wins.

What happens if Castillo accepts a different scholarship and no longer plays football?

Recount points, ignoring all comparisons with C.

A gets 2 points. B gets $2\frac{1}{2}$ points. D gets $1\frac{1}{2}$ points. E gets 0 points.

Byers wins.

Independence of Irrelevant Alternatives Criterion

If candidate X is a winner of an election and in a recount one of the non-winning choices is removed from the ballots, then X should still be a winner of the election.

Method of Pairwise Comparisons violates the Independence of Irrelevant Alternatives Criterion.

What else can happen?

Icelandia State University Hockey Team votes on where to go to dinner- Andrechuk's, Bure's, or Chelio's.

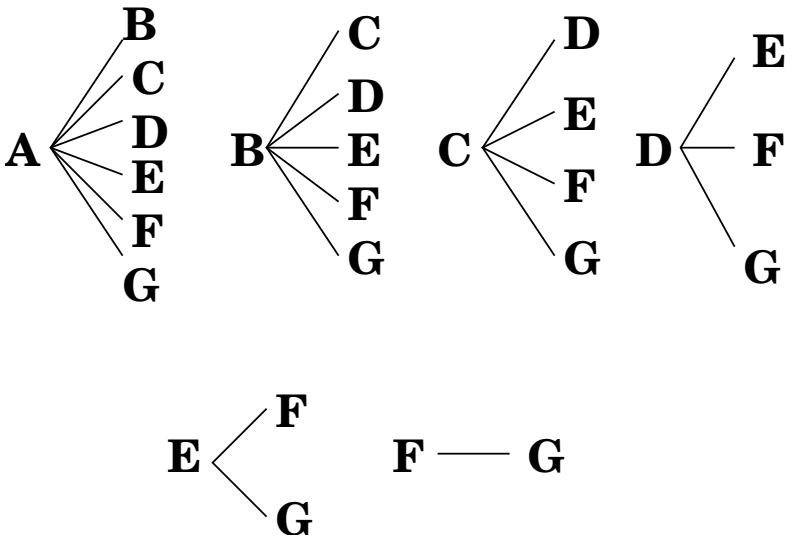
Preference Schedule: Where to eat?

Number of voters	4	2	6	3
First choice	A	B	C	B
Second choice	B	C	A	A
Third choice	C	A	B	C

Note that A beats B, B beats C, and C beats A. Three-way Tie!

Be sure to decide how to break ties before doing the election!

Counting Question: How many games are in a 7-person Rock, Paper, Scissors Tournament?



6 + 5 + 4 + 3 + 2 + 1 = 21

Note: We can use the same process for finding how many pairs we have to consider when doing pairwise comparison.

In general, when N is the number of candidates, the total number of pairs is:

(n-1) + ... + 3 + 2 + 1

Or

$$\frac{n(n-1)}{2}$$

Summary

✧ **Plurality-with-Elimination- Remove the candidate with the least number of first place votes until one candidate has a majority. This candidate wins.**

✧ **Guarentees the winner has a majority of votes**

✧ **Instant runoff- no need to hold a separate election every time a candidate is dropped out of the race.**

✧ **P-W-E Violates the Monotonicity Criterion**

✧ **Used to choose Olympic Host City**

✦ **Method of Pairwise Comparisons- compare candidates one-on-one, wins = 1 point, ties =** $\frac{1}{2}$

✦ **Satisfies the Condorcet, Monotonicity and Majority Criteria**

✦ **Violates the Independence of Irrelevant Alternatives Criterion**

✦ **Possible for everyone to tie**

✦ $\frac{n(n-1)}{2}$ **pairs to compare**