

## Probability 2

Recall:

Outcome

Event

If all outcomes are equally likely:

$$\Pr(E) = \frac{\# \text{ of outcomes leading to } E}{\text{total } \# \text{ of outcomes}}$$

Notice that probability will always be a fraction between 0 and 1. It could also be written as a decimal or a percentage.

Example: In four coin tosses, what is the probability of rolling exactly two heads?

$S = \{HHHH, HHHT, HHHT, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$

Total # of outcomes =  $2 * 2 * 2 * 2 = 16$

$E = \{HHTT, HTHT, HTTH, THHT, THTH, TTHH\}$

# of outcomes leading to E = 6

$\Pr(E) = 6/16 = .375 = 37.5\%$

If E and F are two events that arise from totally separate outcomes, then the probability of E or F happening is the sum of the two probabilities.

$$\Pr(E \text{ or } F) = \Pr(E) + \Pr(F)$$

Note: If the events “overlap” in some outcomes, this won't be true.

Example: Probability of rolling a 5 on one die =  $1/6$

$$\Pr(2) = 1/6$$

$$\Pr(2 \text{ or } 5) = 1/6 + 1/6 = 1/3$$

$$\Pr(1 \text{ or a } 2 \text{ or a } 3 \text{ or a } 4 \text{ or a } 5 \text{ or a } 6) =$$

$$1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 1$$

Sum of probabilities of all possibilities = 1

The probability of E not happening is 1 minus the probability of E happening.

$$\Pr(\text{not } E) = 1 - \Pr(E)$$

## Example: Rolling two dice

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

What is the probability of rolling at least one 3?

What is the probability of rolling no 3s?

## Separate experiments

In two independent experiments, the probability of getting a particular outcome in the first and a particular outcome in the second is the product of the two probabilities.

Event A in first experiment has probability  $\Pr(A)$ .

Event B in second experiment has probability  $\Pr(B)$ .

$$\Pr(A \text{ and } B) = \Pr(A) * \Pr(B)$$

Example: Think of rolling two dice as two independent experiments. What are odds of rolling two 4s?

$$\text{For each roll, } \Pr(4) = 1/6$$

$$\Pr(4 \text{ and } 4) = 1/6 * 1/6 = 1/36$$

Probability of rolling a 5 then a 2?

$$\Pr(5 \text{ and } 2) = 1/6 * 1/6 = 1/36$$

Probability of rolling both a 5 and a 2, in either order?

$$\Pr((5 \text{ and } 2) \text{ or } (2 \text{ and } 5)) = 1/36 + 1/36 = 2/36 = 1/18$$

## Odds

Odds are another way of expressing the likelihood that an event will occur. The odds for an event occurring is the ratio of the number of outcomes leading to the event to the number of outcomes not leading to the event.

Example: Flipping a coin

Odds for getting heads in one toss?

One outcome that leads to this (H)

One outcome that doesn't (T)

Odds are 1:1. Read this "1 to 1".

Odds for getting exactly two heads in three tosses?

Sample space: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Outcomes leading to exactly two heads: {HHT, HTH, THH}

Other outcomes: {HHH, HTT, THT, TTH, TTT}

Odds for getting exactly two heads are 3:5.



Example: Deck of cards

A deck of cards has 4 suits, 13 cards in each suit: A, 2, 3, 4, 5, 6, 7, 8, 9, J, Q, K

Odds for getting a face card on one draw (here, a face card is J, Q, or K)

Odds for drawing a face card are  $12:40 = 3:10$

Odds against? Reverse of odds for.

In example, the odds against drawing a face card are  $10:3$

What are odds for drawing a red face card?

What are odds against drawing a king?

## Converting between odds and probability

If there are  $m$  outcomes that lead to an event, and  $n$  outcomes that don't, there must be  $m+n$  total outcomes.

So if the odds for an event  $E$  are  $m:n$ , then the probability of  $E$  is  $m/(m+n)$ .

Example: probability of drawing a face card =

$$3/(10 + 3) = 3/13 = .2308$$

Say the probability of winning a door prize at a party is  $3/114$ . What are the odds?

3 outcomes win, 114 total, so must be 111 outcomes that aren't winning.

Odds for winning are 3:111

Odds against?

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Odds of rolling a 7 or an 11 with two dice?

Odds against rolling a 7 or an 11?

Probability of rolling a 7 or an 11?

Probability of not rolling a 7 or an 11?