Jefferson's Method

Hamilton's Method is fair until the surplus allocation, and then some states get preferential treatment.

Remove issue of surplus seats.

<u>Jefferson's Method</u>:
Step 1: Modify divisor D so that the lower quotas add up to the number of seats.

Step 2: Apportion to each state its modified lower quota.

<u>How to get D?</u>

1. **Must be smaller than the standard divisor.**

2. **Try it (guess and check)**

    a. **adjust up if total seats > total seats available**
    b. **adjust down if total is still < total seats available**

Example

There are 15 scholarships to be apportioned among 231 English majors, 502 History majors, and 355 Psychology majors.

Total students: 231 + 502 + 355 = 1088

Standard Divisor = 1088/15 = 72.53

Standard quotas:

Major	Std Quota	Lower Quota
English	231/72.53 = 3.185	3
History	502/72.53 = 6.921	6
Psych	355/72.53 = 4.895	4

Total = 13 (2 surplus)

Modify divisor to eliminate surplus seats. (Note: dividing by a smaller number gets a bigger quota).

Use 72:

Major	Std Quota	Lower Quota
English	231/72 = 3.208	3
History	502/72 = 6.97	6
Psych	355/72 = 4.93	4

Total = 13 (2 surplus)

Try again….

Use 71:

Major	Std Quota	Lower Quota
English	231/71 = 3.254	3
History	502/71 = 7.07	7
Psych	355/71 = 5	5

Total = 15, no surplus

Example

Same group, 30 scholarships

Total students: 231 + 502 + 355 = 1088

Standard Divisor = 1088/30 = 36.2667

Major	Std Quota	Lower Quota
English	231/36.267 = 6.369	6
History	502/36.267 = 13.842	13
Psych	355/36.267 = 9.789	9

Total = 28 (2 surplus)

Try 35:

Major	Std Quota	Lower Quota
English	231/35 = 6.6	6
History	502/35 = 14.34	14
Psych	355/35 = 10.14	10

Total = 30 (no surplus)

EXAMPLE

Banana Republic has states Apure, Barinas, Carabobo, and Dolores and 160 seats in the legislature.

Std Divisor:
(3.31 + 2.67 + 1.33 + .69)/160 = .05

State	Size (millions)	Std. Quota .05	Mod. Quota .049	Mod. Quota .0494
Apure	3.31	66.2	67.55	67.004
Barinas	2.67	53.4	54.49	54.04
Carabobo	1.33	26.6	27.14	26.92
Dolores	0.69	13.8	14.08	13.967
Totals	8	158	162	160

Flaw with Jefferson's Method

Jefferson's Method violates the Quota Rule.
(Reminder: A state's apportionment should be either its upper quota or its lower quota. An apportionment method that guarantees that this will happen is said to satisfy the Quota Rule.)

Used for House of Reps until 1832. With 240 seats, NY's standard quota was 38.59 but received 40 seats – Upper Quota violation.

Example 8, Pg 134 – Republic of Parador Allocating 250 legislative seats.

State	Pop	Std Quota	Mod. Quota	Appor-tionment
A	1,646,000	32.92	33.25	33
B	6,936,000	138.72	140.12	140
C	154,000	3.08	3.11	3
D	2,091,000	41.82	42.24	42
E	685,000	13.70	13.84	13
F	988,000	19.76	19.96	19
Total	12,500,000	246	250	250

Note: State B received 140 seats, but his Std Quota was 138.72 (lower quota 138, upper quota 139)

Violates the Quota Rule.

<u>Other Apportionment Methods:</u>

Adams's Method:

Modified Divisor chosen so that upper quotas sum to number of available seats.

Violates Quota Rule (lower quota violation)

Webster's Method:

Modified Divisor chosen so that rounded modified quotas sum to number of available seats.

Violates Quota Rule, but infrequently.

Most Consistently fair

<u>Balinski and Young's Impossibility Theorem</u>

There cannot be a perfect apportionment method. There will either be quota rule violations or paradoxes.

Routing Problems

Find ways to route the delivery of goods and/or services to an assortment of destinations.

- **Newspaper routes**
- **Garbage collection routes**
- **Snowplow routes**
- **Mail carriers**

Key concerns: cost, distance, time
Rules:
if there is "direction of traffic", it must be followed.
if there is no direct way, then a proper route cannot go directly (i.e. no short cuts)
Walking Patrolman:
Traverse every street once
Walking Mail Carrier:
Traverse every street twice

Definitions

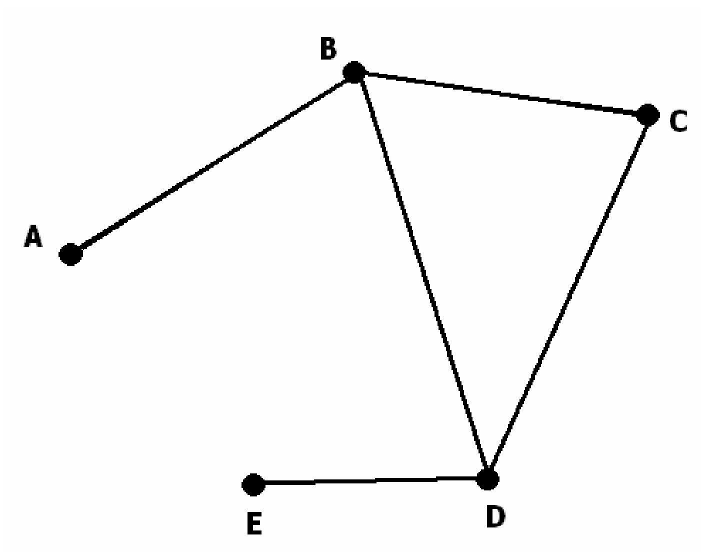
<u>Graph</u>- a structure for describing relationships. (Depiction of a real world routing problem using dots (vertices) and lines (edges) connecting the vertices.)

**<u>Degree of a vertex</u> – the number of edges at
that vertex**

<u>Path</u> – a sequence of vertices with each vertex adjacent to the next (note: a path can cross a vertex more than once, but can only cross an edge once!)

<u>Connected Graph</u> – a graph is connected if any two of its vertices can be joined by a path

<u>Bridge</u> – an edge that connects two components. (if we were to erase the bridge, a connected graph would become disconnected)



Example:

The Pi Appreciation Society meets every Wednesday evening. Two people bring snacks to every meeting. Here is a list of the pairs of those that have brought snacks over the past 9 meetings:

Elle & Vicky
Soley & Hilary
Trent & Matt
Diane & Sean
Vicky & Hilary
Matt & Sean
Elle & Diane
Trent & Sean
Hilary & Matt

Draw a graph that models who has brought snacks together.
Vertices: members
An **edge** connects two members that have brought snacks together

---

Example:

Eve knows Adam, Jacob, and Gina. Adam knows Eve and Gina. Jacob knows Eve, Carla, and Bob. Gina knows Eve and Adam. Carla knows Jacob. Bob knows Jacob. Luke does not know anyone.
Draw a graph modeling this situation.
Vertices: people
An **edge** connects two people that know each other.