Sample Exam 1

Name _____ SOLUTIONS _______
TA Name _________________________

Be sure to use a #2 pencil. Calculators are allowed, but cell phones or palm pilots are NOT acceptable. Please turn cell phones off.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question and mark your answer on the scantron form.

Use the following preference schedule to answer questions #1 - #5:

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>9</th>
<th>2</th>
<th>10</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>C</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>2nd choice</td>
<td>D</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>3rd choice</td>
<td>B</td>
<td>D</td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

1) Using the plurality method, the winner of the election is
   A) A - has 9 first place votes
   B) B - has 0 first place votes
   C) C - has 14 first place votes-- WINNER
   D) D - has 10 first place votes
   E) None of the above.

2) Using the plurality-with-elimination method, the winner of the election is
   A) A - eliminate second because next fewest first place votes
   B) B - eliminate first because no first place votes (no change to other candidates)
   C) C - now has 23 first place votes—WINNER
   D) D - now has 10 first place votes
   E) None of the above.

3) Using the method of pairwise comparisons, the winner of the election is
   A) A - loses to everyone, 0 points
   B) B - beats A, loses to C,D, 1 point
   C) C - wins all, 3 points -- WINNER
   D) D - wins against B and A, loses to C, 2 points
   E) None of the above.

4) Using the extended Borda count ranking method, which candidate comes in 3rd place?
   A) A - 9*1+2*4+10*1+5*2+7*4=65
   B) B - 9*2+2*1+10*3+5*3+7*2=79
   C) C - 9*4+2*3+10*2+5*4+7*3=103
   D) D - 9*3+2*2+10*4+5*1+7*1=83
   E) None of the above.

   **Typo- B is correct answer**

5) Using the recursive plurality method, which candidate comes in second?
   A) A - has 9 first place votes without C
   B) B - has 5 first place votes without C
   C) C - remove as they won first place
   D) D - has 19 first place votes without C--WINNER
   E) None of the above.
6) If candidate A is a winner of an election and, in a reelection, the only changes in the ballots are changes that only favor A, but A does not win the reelection then this is a violation of
   A) The majority criterion
   B) The plurality-with-elimination method
   C) The Independence of Irrelevant Alternatives criterion
   D) The Monotonicity Criterion
   E) None of the above.

7) What is the quota of the weighted voting system?
   \[22 : 10,9,8,5,1\]
   A) 12
   B) 17
   C) 22
   D) 33
   E) None of the above

Refer to the weighted voting system and the Banzhaf definition of power to answer questions #8 - #10:
\[25 : 17,9,8,6\]

8) The winning coalitions for the weighted voting system are
   A) \{17,9,8,6\}, \{17,9,8\}, \{17,9,6\}, \{17,8,6\}, \{17,9\}
   B) \{17,9,8,6\}, \{17,9,8\}, \{17,9,6\}, \{17,8,6\}
   C) \{17,9,8,6\}, \{17,9,8\}, \{17,9,6\}, \{17,8,6\}, \{9,8,6\}, \{17,9\}, \{17,8\}, \{17,6\}
   D) \{17,9,8,6\}, \{17,9,8\}, \{17,9,6\}, \{17,8,6\}, \{9,8,6\}, \{17,9\}, \{17,8\}, \{17,6\}, \{9,6\}, \{8,6\}, \{17\}, \{9\}, \{8\}, \{6\}
   E) \{17,9,8,6\}, \{17,9,8\}, \{17,9,6\}, \{17,8,6\}, \{17,9\}, \{17,8\}

9) Which players in \{17,9,8,6\} are critical?
   A) None of the players are critical.
   B) All of the players are critical.
   C) 17 and 8 are the only critical players.
   D) 17 is the only critical player.
   E) None of the above.

10) What is the Banzhaf power distribution?
    A) \[\frac{6}{11}, \frac{2}{11}, \frac{8}{11}, \frac{2}{11}, \frac{1}{11}\]
    B) \[17, 6; 9: 2; 8: 2; 6: 0\]
    C) \[\frac{1}{2}; \frac{1}{6}; \frac{8}{6}; \frac{1}{6}; \frac{1}{6}\]
    D) \[\frac{3}{5}; \frac{1}{5}; \frac{8}{5}; \frac{1}{5}; \frac{6}{5}; \frac{1}{10}\]
    E) \[\frac{1}{2}; \frac{1}{5}; \frac{8}{5}; \frac{1}{5}; \frac{6}{10}\]

   PLAYER : # TIMES CRITICAL
   17: 6
   9: 2
   8: 2
   6: 0

   TOTAL # CRITICAL: 10
11) In the weighted voting system \([q : 11,8,7,2]\), what is the largest value \(q\) can take on?
A) 15
B) 14
\(\boxed{C) 28 : 11+8+7+2 = 28}\)
D) 2
E) 11

12) In the weighted voting system \([17 : 10,8,7,4,2]\), the total number of possible coalitions, (MEANS DO BANZHAFF) including the empty coalition, is
A) 32 - \(n=5\), formula is \(2^n\) or 32
B) 17
C) 24
D) 35
E) 120

13) In the weighted voting system \([17 : 10,8,7,4,2]\), the total number of possible sequential (MEANS DO SHAPLEY-SHUBIK) coalitions is
A) 32
B) 17
C) 24
D) 35
\(\boxed{E) 120 - n=5\), formula is \(n!\) or \(5!=5*4*3*2*1=120\)}

Refer to the weighted voting system and the Shapley-Shubik definition of power to answer questions #14 - #16: \([10 : 6,3,2]\)

14) Which player in the sequential coalition \(\langle 3,6,2 \rangle\) is pivotal?
A) 6 is the only pivotal player
B) 3 is the only pivotal player
\(\boxed{C) 2 is the only pivotal player}\)
D) All three players are pivotal
E) None of the above.

15) What are the sequential coalitions?
A) \(\langle 6,3,2 \rangle,\langle 6,2,3 \rangle,\langle 3,6,2 \rangle,\langle 3,2,6 \rangle,\langle 2,6,3 \rangle,\langle 2,3,6 \rangle\)
B) \(\langle 6,3,2 \rangle\)
C) \(\langle 6,3,2 \rangle,\langle 3,6,2 \rangle,\langle 2,6,3 \rangle\)
D) \(\langle 6,3,2 \rangle,\langle 6,2,3 \rangle,\langle 3,6,2 \rangle,\langle 3,2,6 \rangle,\langle 2,6,3 \rangle,\langle 2,3,6 \rangle,\langle 6 \rangle\)
E) None of the above.

16) What is the Shapley-Shubik power distribution?
A) \(6 : 2; 3 : 2; 2 : 2\)
B) \(\boxed{6 : \frac{1}{3}; 3 : \frac{1}{3}; 2 : \frac{1}{3}}\)
\(\text{Player : # times pivotal}\)
\(\boxed{6:2}\)
\(3:2\)
\(2:2\)
\(\boxed{\text{Total # pivotal} = 3!=6}\)
C) \(6 : \frac{2}{3}; 3 : \frac{1}{6}; 2 : \frac{1}{6}\)
D) \(6 : \frac{1}{2}; 3 : \frac{1}{2}; 2 : 0\)
E) \(6 : 1; 3 : 0; 2 : 0\)
17) In the weighted voting system \([30 : 30, 25, 4]\) 
A) \(P_1\) is a dictator. 
B) All three players have equal power. 
C) \(P_1\) and \(P_2\) have all the power. 
D) \(P_2\) is a dictator. 
E) None of the above.

Three players agree to divide the items below. Use the players' bids as indicated to answer questions #18-#20.

18) Which of the following is NOT a fair share for player \(B\)? (see blue lines showing \(B\)'s possible fair shares) 
A) /island /police 
B) /hearing /park /soundon /sight /underconstruction 
C) /navigate /6 
D) /UIhelp /hot /medical /navigate /6 
E) None of the above.

19) Under the Method of Markers, what will player \(A\) receive?
A) /island 
B) /UIhelp /hot /medical 
C) /navigate /6 
D) /police /mailboxflagdwn 
E) None of the above.

20) Under the Method of Markers, what items will be leftover?
A) /island /police 
B) /UIhelp /hot /medical 
C) /island 
D) There are no items leftover. 
E) None of the above.
Use the following to answer questions #21-#24.
Aunt Olga dies leaving her children to divide up her antique Singer sewing machine, painting, and Barbie doll collection. They decide to divide up the items using the method of sealed bids. Their bids on each of the items are as follows:

<table>
<thead>
<tr>
<th>Item</th>
<th>Vincent</th>
<th>Valerie</th>
<th>Vanessa</th>
</tr>
</thead>
<tbody>
<tr>
<td>sewing machine</td>
<td>$800</td>
<td>$2,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>painting</td>
<td>$90,000</td>
<td>$99,000</td>
<td>$55,000</td>
</tr>
<tr>
<td>Barbie dolls</td>
<td>$28</td>
<td>$1,000</td>
<td>$1,500</td>
</tr>
<tr>
<td>Total bids</td>
<td>$90,828</td>
<td>$102,000</td>
<td>$61,500</td>
</tr>
<tr>
<td>Fair share</td>
<td>$30,276</td>
<td>$34,000</td>
<td>$20,500</td>
</tr>
<tr>
<td>Item(s) won - fair share</td>
<td>$0-$30,276=-$30,276</td>
<td>$99,000-$34,000=$65,000</td>
<td>$6,500-$20,500=-$14,000</td>
</tr>
</tbody>
</table>

ESTATE: receives $65,000, pays out $30,276 and $14,000 leaving a SURPLUS of -$30,276 + $65,000 - $14,000 = $20,724
One-third of the surplus is: $6,908

21) Who will receive the Singer sewing machine?
   A) Vincent
   B) Valerie
   C) Vanessa
   D) Valerie and Vanessa
   E) None of the above.

22) What is Vincent’s fair share?
   A) $90,825
   B) $30,276
   C) $20,724
   D) $37,184
   E) None of the above.

23) After the final allocation, how much money will Valerie have to pay in?
   A) $65,000
   B) $99,000
   C) $58,092 (paid estate $65,000, but got $6,908 back from surplus)
   D) $34,000
   E) None of the above

24) After the final allocation, how much money will Vanessa receive?
   A) $14,000
   B) $20,500
   C) $61,500
   D) $20,908 ($14,000 from the estate, plus the surplus of $6,908)
   E) None of the above