

Reassessing L1 Tolerance

L1: I can determine input tolerance (δ) from a given function and output tolerance (ϵ).

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about *HOW* you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
 - (a) not understanding a concept;
 - (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
 - (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
 - (d) not knowing how to start or formulate an approach to the problem;
 - (e) others?

Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

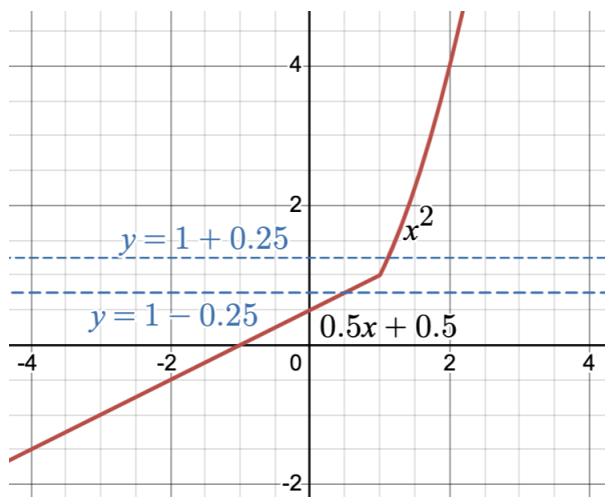
4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 1

Theory and Example:

Consider the piecewise-defined function, $k(x)$ below.

$$k(x) = \begin{cases} 0.5x + 0.5 & x < 1 \\ x^2 & x \geq 1 \end{cases}$$



1. What is the limit as x approaches 1? $L = 1$
2. **Algebraically** find the largest interval of x -values around $x = 1$ that will ensure the function values lie within 0.25 of the limit. Use **2** decimal places.

Add the horizontal lines $y = 1 + 0.25 = 1.25$ and $y = 1 - 0.25 = 0.75$ to the graph. Notice where they intersect the function.

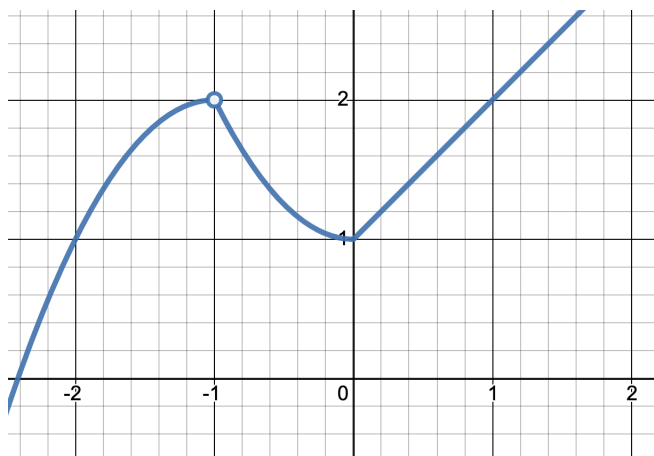
To find the lower bound, set $0.5x + 0.5 = 0.75$ and solve for x . We get $x = 0.5$.

To find the upper bound, set $x^2 = 1.25$. When we take the square root, we get TWO solutions: $\pm\sqrt{1.25}$, or $x = \pm 1.11803$. We only need one of these solutions, so use the graph to help you decide- the intersection we need is to the right of 1, so we want the positive solution. Rounding can be tricky- because this is tolerance, we need to “round in” to make sure our interval isn’t too large. For example $1.12^2 = 1.2544 > 1.25$, so doing normal rounding to two decimal places doesn’t work. We should pick 1.11, and to double check, $1.11^2 = 1.2321 < 1.25$.

$$\underline{0.5} < x < \underline{1.11}$$

Consider the piecewise-defined function, $j(x)$ below.

$$j(x) = \begin{cases} -(x+1)^2 + 2 & x < -1 \\ x^2 + 1 & -1 < x < 0 \\ x + 1 & x \geq 0 \end{cases}$$



Algebraically find the interval of x -values around $x = -1$ that will ensure the function values lie within 0.25 of 2. (In other words, let $L = 2$ and $\epsilon = 0.25$.) Use **3** decimal places.

Favorite Mistakes

- Not calculating the largest and smallest acceptable output values ($L + \epsilon$ and $L - \epsilon$)
- Solving graphically or estimating to get the interval of x values. Follow the procedure outlined above and use algebra techniques.
- Setting up equations/inequalities without referencing the graph. (Setting up more than just 2 equations.) If you are looking around $x = -1$, to find the lower bound you want $x < -1$ so use the part of $j(x)$ for those x values, namely $-(x+1)^2 + 2$. For the upper bound, $x > -1$ and when $x > -1$, $j(x) = x^2 + 1$. For x near 0, use $x^2 + 1$ for the lower bound and $x + 1$ for the upper bound.
- Not using the graph to determine if you should use 2.25, 1.75, 0.75 or 1.25. Around $x = -1$, only $y = 1.75$ intersects the function, so you would never use $j(x) = 2.25$. Likewise around $x = 0$, only $y = 1.25$ intersects the function.
- Selecting the wrong root/forgetting that $x^2 = c$ usually has two solutions. $\sqrt{0.5} = \pm 0.5$ so when solving $x + 1 = \pm\sqrt{0.5}$ you need to choose between $x = -0.5$ and $x = -1.5$. You can use the graph to help see which number makes sense, or you can think about the interval of x values where you are investigating... (in this case, x close to -1, but less than -1.)

Prepare for Reassessment

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks “I used to think _____ but now I think _____ because I learned _____.”

1. Let $f(x) = \begin{cases} (x+2)^2 + 1 & x \leq -2 \\ -x^2 + 5 & x > -2 \end{cases}$.

Algebraically find the largest interval of x -values around $x = -2$ that will ensure the function values lie within 0.25 of 1. (In other words, let $L = 1$ and $\epsilon = 0.25$.) Use **3** decimal places.

I strongly recommend graphing the function and the $y = L + \epsilon$ and $y = L - \epsilon$ lines to help you.

2. Let $f(x) = e^x$, consider the point $(0, 1)$. Select all intervals that guarantee that $f(x)$ is within 0.1 of 1.

(a) $0 < x < 0.8$

(b) $-0.1 < x < 0.1$

(c) $-0.105 < x < 0.0954$

(d) $-0.1054 < x < 0.095$

I strongly recommend you find the largest interval of x values rather than just checking the 4 options above.