Reassessing I4

I4: I can use shortcuts to compute antiderivatives for sums, constant multiples, and power, polynomial, trig, exponential functions, and $\frac{1}{x}$.

Where topic was first introduced: Module 13

Reminder of shortcut rules:

Function	**General Antiderivative (Indefinite Integral)**
x^n	$\frac{1}{n+1}x^{n+1} + C, n \neq -1$
$\sin(kx)$	$\frac{-1}{k}\cos(kx) + C$
$\cos(kx)$	$\frac{1}{k}\sin(kx) + C$
$\sec^2(kx)$	$\frac{1}{k}\tan(kx) + C$
e^{kx}	$\frac{1}{k}e^{x} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$

Favorite Mistakes:

- Mixing up the derivative and antiderivatives for sine and cosine (which one has the negative sign???)
- +C
- Absolute value signs on the natural log

How do I know if I'm doing this right?

- Take the derivative of your answer. This is usually a good way to check your work on quizzes and exams.
- If you have access to technology, you can use an integral calculator like Symbolab.
- You can use Desmos- make up some bounds and evaluate definite integrals (type "int" into Desmos to get a definite integral.)
- Use Desmos to graph the integrand, and graph the derivative of the answer you found. (i.e. let Desmos do the derivative.)

Practice

(a) $\int \frac{1}{x^{0.5}}\, dx=$ $\int x^{-0.5}\, dx = \frac{x^{0.5}}{0.5} + C = 2x^{\frac{1}{2}} + C$

$= 2\sqrt{x} + C$

(b) $\int \cos(x)\ dx=$ $\sin(x) + C$

(c) $\int \sin(x)\ dx=$ $-\cos(x) + C$

common mistake - omitting absolute value

(d) $\int \frac{1}{x}\, dx=$ $\ln|x| + C$

§6.2 6-21 & 43-56

(e) $\int e^{3x}\ dx=$ $\frac{1}{3}e^{3x} + C$

1. If $p'(x) = q(x)$, write a statement involving an integral sign giving the relationship between $p(x)$ and $q(x)$.
2. If $u'(x) = v(x)$, write a statement involving an integral sign giving the relationship between $u(x)$ and $v(x)$.
3. Which of (I)–(V) are antiderivatives of $f(x) = e^{x/2}$?

   I. $e^{x/2}$ II. $2e^{x/2}$ III. $2e^{(1+x)/2}$
   IV. $2e^{x/2}+1$ V. $e^{x^2/4}$
4. Which of (I)–(V) are antiderivatives of $f(x) = 1/x$?

   I. $\ln x$ II. $-1/x^2$ III. $\ln x + \ln 3$
   IV. $\ln(2x)$ V. $\ln(x+1)$
5. Which of (I)–(V) are antiderivatives of

   $$f(x) = 2\sin x \cos x?$$

   I. $-2\sin x\cos x$ II. $2\cos^2 x - 2\sin^2 x$
   III. $\sin^2 x$ IV. $-\cos^2 x$
   V. $2\sin^2 x + \cos^2 x$

■ In Exercises 6–21, find an antiderivative.

6. $f(x) = 5$
7. $f(t) = 5t$
8. $f(x) = x^2$
9. $g(t) = t^2 + t$
10. $g(z) = \sqrt{z}$
11. $h(z) = \frac{1}{z}$
12. $r(t) = \frac{1}{t^2}$
13. $h(t) = \cos t$
14. $g(z) = \frac{1}{z^3}$
15. $q(y) = y^4 + \frac{1}{y}$
16. $f(z) = e^z$
17. $g(t) = \sin t$
18. $f(t) = 2t^2 + 3t^3 + 4t^4$
19. $f(x) = 5x - \sqrt{x}$
20. $f(t) = \frac{t^2+1}{t}$
21. $p(t) = t^3 - \frac{t^2}{2} - t$

■ In Exercises 22–33, find the general antiderivative.

22. $f(t) = 6t$
23. $h(x) = x^3 - x$
24. $f(x) = x^2 - 4x + 7$
25. $g(t) = \sqrt{t}$
26. $r(t) = t^3 + 5t - 1$
27. $f(z) = z + e^z$
28. $g(x) = \sin x + \cos x$
29. $h(x) = 4x^3 - 7$
30. $g(x) = \frac{5}{x^3}$
31. $p(t) = 2 + \sin t$
32. $p(t) = \frac{1}{\sqrt{t}}$
33. $h(t) = \frac{7}{\cos^2 t}$

■ In Exercises 34–41, find an antiderivative $F(x)$ with $F'(x) = f(x)$ and $F(0) = 0$. Is there only one possible solution?

34. $f(x) = 3$
35. $f(x) = 2x$
36. $f(x) = -7x$
37. $f(x) = 2 + 4x + 5x^2$

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■ In Exercises 43–56, find the indefinite integrals.

43. $\int (5x+7)\, dx$
44. $\int \left(4t + \frac{1}{t}\right) dt$
45. $\int (2+\cos t)\, dt$
46. $\int 7e^x\, dx$
47. $\int (3e^x + 2\sin x)\, dx$
48. $\int (4e^x - 3\sin x)\, dx$
49. $\int \left(5x^2 + 2\sqrt{x}\right) dx$
50. $\int (x+3)^2\, dx$
51. $\int \frac{8}{\sqrt{x}}\, dx$
52. $\int \left(\frac{3}{t} - \frac{2}{t^2}\right) dt$
53. $\int (e^x + 5)\, dx$
54. $\int t^3(t^2+1)\, dt$
55. $\int \left(\sqrt{x^3} - \frac{2}{x}\right) dx$
56. $\int \left(\frac{x+1}{x}\right) dx$