Reassessing I2

I2: I can interpret the meaning of an integral in context and recognize when an integral would be suitable to model a real life situation.

Where topic was first introduced: Module 12

Favorite Mistakes:

- Not using units in your answer.
- Just restating the question (Yes, an integral will give us oil.) rather than explaining WHY an integral/Riemann sum works theoretically. Focus on the fact that integrals work great in situations where we have multiplication.
- Accounting for and writing dx (or dt etc).
- When doing areas, make sure you draw the rectangle and get the correct order for the subtraction (top-bottom).

Examples:

1. The amount of energy used by a lightbulb over a given time can be measured by the equation energy (in joules) = power (in watts) × time (in seconds). But this equation only works if the amount of energy used is **constant!** If your lightbulb can be dimmed and the amount of power used is **non-constant** and represented by $p(t)$, what is the equation for the total amount of energy used by the lightbulb from time $t = 0$ to $t = T$? Explain your equation.

$$\text{Energy } = \int_{t=0}^{T} p(t)dt$$

Energy is typically represented as power multiplied by time when power is constant. However, in this case, power is non-constant, and depends on the time. Because t is the independent variable, we replace the multiplication with an integral of $p(t)$ with respect to t. Looking at the units on $p(t)$ we have watts, and t is measured in time, so the integral gives us watts × time= joules, which are the units for energy.

2. Help Hugh Manatee find the following quantities! Circle the context below if an integral would be a suitable way to find the requested quantity. If the integral is suitable, also write the definite integral AND the units for what the integral measures. if an integral is NOT suitable, write "For the love of Hugh Manatee, alas, I cannot help thee!"

   (a) $p(x)$ measures the amount of dust that has accumulated on the solar panels in grams. Hugh wants to find how fast the dust is accumulating in the first 20 minutes. "For the love of Hugh Manatee, alas, I cannot help thee!" You want a derivative in this case, not an integral! The units of an integral would be grams $\times$ minutes, which doesn't make sense for the amount of dust, which would be measured in just grams.

(b) $f(t)$ measures velocity in meters per second with respect time in seconds. Hugh wants to find distance traveled between times 3 and 7.

The distance traveled would be $\int_3^7 |f(t)| dt$. (Without the absolute values we might be measuring displacement instead.) The units would be meters per second $\times$ seconds = meters, which is the unit for measuring distance.

(c) $m(x)$ measures the density in kilograms per meter with respect to distance in meters. Hugh wants to find mass of the object that is 3 meters long. mass=$\int_0^3 m(x)dx$. Unit check: kg/m*m=kg. Note that the integral of density is mass, but integrating mass won't give us mass, but a mass-meter, whatever that is....

(d) $a(t)$ measures the velocity in meters per second. Hugh wants to find the acceleration at time 5. "For the love of Hugh Manatee, alas, I cannot help thee!"' You want a derivative in this case, not an integral! An integral of velocity would give position, units: meters per second $\times$ second = meters, rather than meters per second per second for acceleration.

(e) $R(a)$ measures the rate that oil is leaking out of a tanker in gallons per hour. Hugh wants to find the total amount of oil that has leaked between times 0 and 4 hours. total oil: $\int_0^4 R(a)da$. Units: gallons/hr*hr=gallons

Prepare for Revision:

1. $p(x)$ measures the amount of coal that is produced in the US annually, measured in billions of tons per year.

   What quantity would $\int_3^6 p(x)dx$ measure? Include units.

2. $f(t)$ measures velocity in miles per hour. Would an integral be a suitable strategy for measuring acceleration? If not, what would an integral measure? Include units.

3. $m(x)$ measures the density in kilograms per meter with respect to distance in meters. What would an integral $\int_a^b m(x)dx$ measure? Include units.

4. $\int_1^4 R(t)dt$ measures the amount of oil collected between times 1 day and 4 days in thousands of gallons.What does $R(t)$ measure? Include units.

5. Set up an integral that gives the area between $f(x) = \ln(x) + 10$ and $g(x) = 0.5 + 1$ between $x = 1$ and $x = 2$.

6. Set up an integral that gives the area enclosed by $f(x) = -x^2 + 20$ and $g(x) = x^2 + 4$.